Fundamentals
M1
MATHEMATICS

Rev.-ID: 1JAN2014
Author: HiT
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©LTT Release: May. 21, 2014

EASA Part-66
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Training Manual

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M1 MATHEMATICS

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M1.1 ARITHMETIC

Introduction

General

Just as studying a new language begins with learning basic words, the study of mathematics begins with arithmetic, its most basic branch. Arithmetic uses real and non−negative numbers, which are also known as counting numbers, and consist of only four operations:

- addition
- subtraction
- multiplication
- division.

While you have been using arithmetic since childhood, a review of its terms and operations will make learning the more difficult mathematical concepts much easier.

Digits

Numbers are represented by symbols which are called digits. There are nine digits which are 1, 2, 3, 4, 5, 6, 7, 8 and 9. We also use the symbol 0 (i.e. zero) where no digits exists. Digits and zero may be combined together to represent any number.

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Addition

General

The process of finding the total of two or more numbers is called addition. This operation is indicated by the plus (+) symbol. When numbers, called summands, are combined by addition, the resulting total is called the sum.

When adding whole numbers whose total is more than nine, it is necessary to arrange the numbers in columns so that the last digit of each number is in the same column. The ones column contains the values zero through nine, the tens column contains multiples of ten, up to ninety, and the hundreds column consists of multiples of one hundred.

Example:

	hundreds	tens	ones	
		7	8	(first summand)
+	2	4	3	(second summand)
+	4	6	2	(third summand)
	7	8	3	(sum)

To add the sum of the above, first add the ones column: 8 and 3 and 2 makes 13. Place the 3 in the ones column of the answer and carry the 1 forward to the tens column.

Adding this we add: 1 and 7 and 4 and 6 is 18. Place the 8 in the tens column of the answer and carry the 1 forward to the hundreds column which we now add.

1 and 2 and 4 is 7. Place the 7 in the hundreds column of the answer. We see that the answer (sum) to the addition is 783.

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Subtraction

General

The process of finding the difference between two numbers is known as subtraction and is indicated by the minus (–) sign. Subtraction is accomplished by taking the quantity of one number away from another number. The number which is being subtracted is known as the subtrahend (smaller number), and the number from which the quantity is taken is known as the minuend (larger number).

To find the difference of two numbers, arrange them in the same manner used for addition. With the minuend on top and the subtrahend on the bottom, align the vertical columns so the last digits are in the same column. Beginning at the right, subtract the subtrahend from the minuend. Repeat this for each column.

Example:

	hundreds	tens	ones	
	4	4	3	(minuend)
–	2	6	2	(subtrahend)
	1	8	1	(difference)

Place 262 under 443. 2 from 3 leaves 1. write 1 in the ones column of the answer. 6 from 4 is clearly impossible, so the 4 is increased in value to 14 by taking 1 from the hundreds column leaving 3. 14 from 6 leaves 8. Write 8 in the tens column. Finally, 3 from 2 in the hundreds columns leaves 1.

To check a subtraction problem, you can add the difference to the subtrahend to find the minuend.

There are two methods by which subtraction can be performed.

Consider **15 – 8 = 7**

Method 1

Take 8 from 15. We have 7 left. This method is the most common method.

Another way is also used to write the calculation, especially when the numbers are greater.

```
  15
- 8
  7
```

Method 2

Add various numbers to the 8. When the result is 15, you found the correct number.

If we add 8 to 7 then we obtain 15. 7 is therefore the difference between 15 and 8.

This method is a kind of “try and error“.

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Multiplication

General

Multiplication is a special form of repetitive addition.

When a given number is added to itself a specified number of times, the process is called multiplication. The sum of 4 + 4 + 4 = 12 is expressed by multiplication as 4 × 3 = 12.

The numbers 4 and 3 are called factors and the answer 12, represents the product.

The number multiplied (4) is called the multiplicand, and the multiplier represents the number of times the multiplicand is added to itself, in this case 3.

Sign

Multiplication is typically indicated by the symbol ×.

In many countries the middle dot is used, which is also called interpunct. An operation with the middle dot looks like this: 2 · 2 = 4

In some cases (e.g. algebra and/or brackets), it is denoted by the lack of any other operation sign.

Examples:

5 × 3 = 15

5 · 3 = 15

5ab = 5 · a · b

Sequence

When multiplying, the order in which numbers are multiplied does not change the product.

Example:

3 × 4 = 12

4 × 3 = 12

Procedure

Like addition and subtraction, when multiplying large numbers it is important that they have to be aligned vertically.

When multiplying numbers greater than nine, multiply the multiplicand by each digit in the multiplier. Once all multiplicands are used as a multiplier, the products of each multiplication operation are added to arrive at a total product.

Example:

Multiplicand is 532

Multiplier is 24

Write:

532 × 24

Calculation:

532 ×	**24**	
1064		(first partial product)
+ 2128		(second partial product)
12768		**(product)**

The product is 12 768.

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Division

General

Just as subtraction is the reverse of addition, division is the reverse of multiplication. Division is a means of finding out how many times a number is contained in another number. The number divided is called dividend, the number you are dividing by is the divisor, and the result is the quotient.

Remainder

With some division problems, the quotient may include a remainder. A remainder represents that portion of the dividend that cannot be divided by the divisor.

Sign

Division is indicated by the use of the division signs (÷) , (:), (/) with the dividend to the left and the divisor to the right of the sign, or with the dividend inside the sign and the divisor to the left.

Large Quantities

The process of dividing large quantities is performed by breaking the problem down into a series of operations, each resulting in a single digit quotient.

This is best illustrated by an example.

Example:

Let us divide 416 by 8.

Dividend is 416

Divisor is 8

Write:

416 ÷ 8 = 52

or

416 : 8 = 52

Calculation:

416 : 8 = 52

40

16

16

0

1. **4** : **8** = Not possible
2. **41** : **8** = **5** Remainder **1**
3. **16** : **8** = **2** Remainder 0

1. Take the first digit, the **4.**
   Try to divide **4** by **8**. This is not possible.
2. Take the first two digits, the **41**.
   Try to divide **41** by **8**. This is **5** and the remainder is **1**.
3. Take the remainder 1 and the third digit, the **6**.
   Combine to **16**.
   Try to divide **16** by **8**. This is **2** and the remainder is 0.
4. Take the first digit you got (in step 2) and the second digit you got (in step 3). This is **52**, and this is the result.

The quotient is 52.

Check

To check a division problem for accuracy, multiply the quotient by the divisor and then add the remainder (if any).

If the operation is carried out properly, the result equals the dividend.

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Priority Rules

Operations

When a term is to be calculated, there is a priority rule for operations.

Multiplications and divisions must be done first.

Then additions and subtractions are done.

Example:

$$5 + 3 \times 2 = 11$$

If you do the addition first and then multiply by 2, you get a wrong result.

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Signed Numbers

Adding Signed Numbers

When adding two or more numbers with the same sign, ignore the sign and find the sum of the values and then place the common sign in front of the answer. In other words, adding two or more positive numbers always results in a positive sum, whereas adding two or more negative numbers results in a negative sum.

When adding a positive and negative number, find the difference between the two numbers and apply (+ or –) of the larger number. In other words, adding negative number is the same as subtracting a positive number. The result of adding or subtracting signed numbers is called algebraic sum of those numbers.

Example:

25 + (–15) = 10
25 - 15 = 10

Subtracting Signed Numbers

When subtracting numbers with different signs, change the operation sign to plus and change the sign of the subtrahend. Once this is done, proceed as you do in addition. For example +3 – (–4) is the same as +3 + 4. There is no difference if the subtrahend is larger than the minuend, since the operation is done as though the two quantities are added.

Example:

Subtract 48 from –216

Step 1: Set up the subtraction problem –216 – 48

Step 2: Change the operation sign to a plus sign and change the sign of the subtrahend. Now add:

(–216) + (–48) = –264

Multiplying Signed Numbers

Multiplication of signed numbers is accomplished in the same manner as multiplication of any other number. However, after multiplying, the product must be given a sign. There are three rules to follow when determining a products sign.

1. The product of two positive numbers is always positive.
   Example: 6 x 2 = 12
2. The product of two negative numbers is always positive.
   Example: (–6) x (–2) = 12
3. The product of a positive and a negative number is always negative.
   Examples: 6 x (–2) = –12
   (–6) x +2 = –12

Dividing Signed Numbers

Like multiplying signed numbers, division of signed numbers is accomplished in the same manner as dividing any other number. The sign of the quotient is determined using the rules identical to those used in multiplication.

Examples:

12 ÷ 3 = 4
12 ÷ (–3) = –4
(–12) ÷ 3 = –4
(–12) ÷ (–3) = 4

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Weight & Mass

General

Weight is defined as the gravitational pull of the earth on a given body. This is a force.

The direction of this force is regarded toward the geometrical centre of the earth.

Physicists are very careful to distinguish between “mass & weight”. But in normal life, people do not distinguish. We mean mass but say weight. On the market, we buy an amount of potatoes with an exact weight.

Due to this fact in the following we speak correctly about the mass.

System of Measurement

The system of measurement is based mainly on the International System of Units, usually abbreviated as SI (french: Système international d’unités).

Unit

The kilogramme (kg) is the SI unit for mass.

In 1889 the kilogramme was defined by a platinum – iridium alloy cylinder of 39 mm in diameter and 39 mm in height. This original or prototype kilogramme is stored in a safe near Paris the French capital.

Using the decimal system, based on number of 10 we are able to calculate bigger and smaller values. In accordance to the decimal system 1 kg consists of 1000 g (gram) and 1 g in turn consists of 1000 mg (milligramme).

Multiplying 1 kg by 1000 we reach 1 Mg (mega gram) usually called 1 metric ton.

In daily life we use the units between gram and ton.



Figure 1 Decimal System Kilogramme

Prefix	Symbol	Factor
tera	T	1 000 000 000 000
giga	G	1 000 000 000
mega	M	1 000 000
kilo	k	1 000
hecto	h	100
deca	da	10
deci	d	Divide by 10
centi	c	Divide by 100
milli	m	Divide by 1 000
micro	µ	Divide by 1 000 000
nano	n	Divide by 1 000 000 000

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Measures

General

Measures are used to give an exact impression of a distance.

This may be the length of a street or the length of an object.

Unit

The metre (m) is the SI unit for length.

In 1875 the original or prototype metre was defined as the 10millions part of 1/4 of an earth meridian. A meridian quadrant is approximately the distance from the geographic pole to the equator. Also the prototype metre is made of a platinum – iridium alloy and stored in France.

Since 1983 the scholars use natural constants to define some basic units. Therefore the metre is defined as the distance travelled by light in a vacuum in $\frac{1}{299792458}$ of a second. The speed of light is a natural constant and defined with 299 792 458 metre per second.

Natural constants remain unchanged and can be reproduced all over the world, at least theoretically.

Using the decimal system, based on number of 10 we are also able to calculate bigger and smaller values. In accordance to the decimal system 1 m consists of 10 dm or 100 cm or 1000 mm.
Multiplying 1 m by 1000 we reach 1 km. In daily life we use the values between kilometre and millimetre.

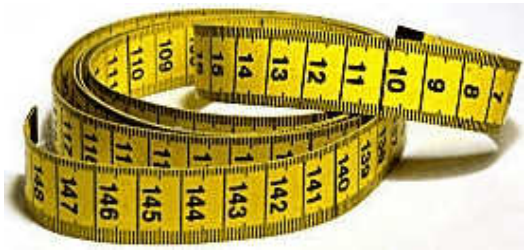


Figure 2 Decimal System Metric

Prefix	Symbol	Factor
tera	T	1 000 000 000 000
giga	G	1 000 000 000
mega	M	1 000 000
kilo	k	1 000
hecto	h	100
deca	da	10
deci	d	Divide by 10
centi	c	Divide by 100
milli	m	Divide by 1 000
micro	μ	Divide by 1 000 000
nano	n	Divide by 1 000 000 000

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Conversion of Units

General

As already mentioned, the majority of the countries in Europe and most of the countries in the world are using the SI units to define the basic units and their derived units.

The remaining countries e.g. Great Britain and the USA are using an older system called the imperial system. The imperial system uses units which are historically grown and developed over centuries (e.g. feet, inches, yards, pounds and gallons). It is still used in aviation until today.

Length

1 in (inch)	=	25.4 mm
1 m	=	39.37 in or 3.281 ft
1 ft (foot)	=	12 in or 0.3048 m
1 yd (yard)	=	3 ft or 36 in or 0.9144 m
1 km	=	0.621 miles
1 mile	=	1760yd = 5280 ft or 1.61 km
1 n.m.	=	1.151 miles or 1.852 km

Area

1 m^2	=	10.000 cm^2
1 m^2	=	10.76 ft^2
1 acre	=	4840yd^2 = 4046.87m^2
1 yd^2	=	9 ft^2
1 ft^2	=	144 in^2 or 0.0929 m^2
1 in^2	=	6.452 cm^2

Volume

1 m^3	=	1 000 dm^3 = 1000 litre
1 ft^3	=	1728 in^3 = 0.0283 m^3
1 litre	=	1000 cm^3 = 1.0576 qt (US)
1 qt (US quart)	=	0.8327 qt (UK) = 0.9464 litre
1 gal (US gallon)	=	8 pints (US)
1 gal (UK)	=	4.546 litres
1 gal (US)	=	3.785 litres

Mass

1 metric ton	=	1000 kg
1 kg	=	1000 g
1 lb	=	0.4536 kg = 453.6 g
1 lb	=	16 oz
1 oz	=	28.3495 g

Velocity

1 km/h	=	3.6 m/s
1 m/s	=	3.281 ft/s
1 mph	=	1.47 ft/s
1 mph	=	1.61 km/h
1 knot	=	1.688 ft/s
1 knot	=	1.151 mph
1 knot	=	1.852 km/h

Force & Weight

1 N	=	0.2248 lbf
1 N	=	3.5969 ozf

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Torque

1 daNm	=	10 Nm
1 Nm	=	8.851 lbf in
1 lbf ft	=	12 lbf in

Power

1 HP (metric)	=	735.4988 W
1 HP (UK)	=	745.6999 W
1 HP (metric)	=	0.9863 HP (UK)
1 HP (metric)	=	542.4760 lbf ft/s
1 HP (UK)	=	550 lbf ft/s
1 W	=	0.738 ft lb/s
1 Btu/h	=	0.293 W

Pressure

1 Pa	=	0.000145 lb/in^2
1 bar	=	100 000 Pa = 1000 hPa
1 bar	=	14.5038 PSI
1 bar	=	750.0638 mmHg
1 bar	=	29.53 inHg
1 atm	=	29.92 inHg
1 PSI	=	689 kPa

Energy

1J	=	0.738 ft lb
1 cal	=	4.186 J
1 Btu	=	252 cal

Time

1 year	=	365 days
1 day	=	24 h = 1440 min

Temperature (conversion formula may required)

Δ1°C	=	Δ1 K (Kelvin)
1 °C	=	33.8 °F (Fahrenheit)
1 °F	=	– 17.22 °C
1 °C	=	493.47 °R (Rankine)
1 °R	=	– 272.59 °C or – 458.67 °F or 0.5556 K
0 K	=	– 273.15 °C or – 459.67 °F or 0 °R

$$°F = \left(°C \times \frac{9}{5}\right) + 32$$

$$°C = (°F - 32) \times \frac{5}{9}$$

Other Useful Data

1 litre water	=	1 kg
1 pint water	=	1 lb

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Factors and Multiples

Factors

Factors can be used to count the units.

If something weights 5 x 1kg so 5 is a factor.

The calculation is a multiplication.

Sometimes short expressions for factors are used for units. These are called prefixes. This is used to shorten long expressions like 80 000 metres to 80 km.

The k is for kilo and means the factor 1 000.

Kilo, hecto, deca, centi and milli are the most common metric prefixes in daily use. All other show either values too big or too small for daily use.

In the metric world metric prefixes are used.

They all are based on the factor 10.

Metric Prefixes

Prefix	Symbol	Factor
tera	T	1 000 000 000 000
giga	G	1 000 000 000
mega	M	1 000 000
kilo	k	1 000
hecto	h	100
deca	da	10
deci	d	Divide by 10
centi	c	Divide by 100
milli	m	Divide by 1 000
micro	µ	Divide by 1 000 000
nano	n	Divide by 1 000 000 000

Multiples

Multiples are the simple product of a multiplication.

A football team has 11 sportsmen on the field.

During the ceremonies at the beginning of Olympic Games you may see many teams the same time. So you have 44, 99 or even more sportsmen.

This is a multiple. 99 sportsmen on the lawn, that is 9 times a team.

Dividing Multiples

When you divide a multiple by 2, 3, 4 etc. you will get a natural number.

When you see multiple sportsmen on the field and you divide this number by the sportsmen in a team, you get the number of teams.

For example 99 sportsmen divided by 11 sportsmen for a team has a result of 9 teams.

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Common Fractions

Introduction

A common fraction represents a portion or part of a quantity.

For example, if a number is divided into three equal parts, each part is one–third ($\frac{1}{3}$) of the number.

A fraction consists of two numbers, one above and one below a line, or fraction bar. The fraction bar indicates division of the top number, or numerator, by the bottom number, or denominator.

For example, the fraction $\frac{3}{4}$ indicates that three is divided by four to find the decimal equivalent of 0.75.

When a fractions numerator is smaller than the denominator, the fraction is called a proper fraction. A proper fraction is always less than 1. If the numerator is larger than the denominator, the fraction is called an improper fraction. In this situation the fraction is greater than 1. If the numerator and the denominator are identical, the fraction is equal to 1.

A mixed number is the combination of a whole number and a proper fraction.

Mixed numbers are expressed as $1\frac{5}{8}$ and $29\frac{9}{16}$ and are typically used in place of improper fractions. The numerator and denominator of a fraction can be changed without changing the fractions value. A mixed number can be converted into an improper fraction and vice versa.

Example 1:

Express the mixed number $8\frac{2}{3}$ as a proper fraction.

$$8\frac{2}{3} = \frac{(8 \times 3) + 2}{3} = \frac{24 + 2}{3} = \frac{26}{3}$$

Example 2:

Express the improper fraction $\frac{27}{4}$ as a mixed number.

$$\frac{27}{4} = \frac{24 + 3}{4} = \frac{24}{4} + \frac{3}{4} = 6\frac{3}{4}$$

(since $27 \div 4 = 6$ remainder 3)

Lowest Terms

A fraction is said to be in its lowest terms when it is impossible to find a number which will divide exactly into both its numerator and denominator.

The fractions $\frac{5}{7}$ and $\frac{11}{19}$ are both in their lowest terms but the fraction $\frac{6}{10}$ is not in its lowest terms because it can be reduced to $\frac{3}{5}$ by dividing top and bottom numbers by 2.

Example:

Reduce $\frac{21}{35}$ to its lowest terms.

$$\frac{21}{35} = \frac{21 \div 7}{35 \div 7} = \frac{3}{5}$$

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Handling Fractions

Comparing the Size of Fractions

When the values of two or more fractions are to be compared, express each of the fractions with the same denominator. This common denominator should be the lowest common denominator of the denominator of the fractions to be compared. It is sometimes called LCM (lowest common denominator).

Example:

Arrange the fractions $\frac{5}{6}$, $\frac{8}{9}$ and $\frac{7}{8}$ in order of size beginning with the smallest.

The LCM of the denominators 6, 8, and 9 is 72.

$\frac{5}{6}$ is equivalent to $\frac{5 \times 12}{6 \times 12} = \frac{60}{72}$

$\frac{8}{9}$ is equivalent to $\frac{8 \times 8}{9 \times 8} = \frac{64}{72}$

$\frac{7}{8}$ is equivalent to $\frac{7 \times 9}{8 \times 9} = \frac{63}{72}$

Because all the fractions have been expressed with the same denominator all that we need to do is to compare the numerators.

Therefore the order of size is $\frac{60}{72}, \frac{63}{72}, \frac{64}{72}$ or $\frac{5}{6}, \frac{7}{8}, \frac{8}{9}$.

Adding & Subtracting Fractions

Two fractions which have the same denominator can be added together by adding their numerators. Thus

$$\frac{3}{11} + \frac{5}{11} = \frac{(3 + 5)}{11} = \frac{8}{11}$$

When two fractions have different denominators they cannot be added together directly. However, if we express the fractions with the same denominator they can be added.

Example:

Add $\frac{2}{5}$ and $\frac{3}{7}$

The lowest common denominator of 5 and 7 is 35.

$$\frac{2}{5} + \frac{3}{7} = \frac{2 \times 7}{5 \times 7} + \frac{3 \times 5}{7 \times 5} = \frac{14}{35} + \frac{15}{35}$$

$$= \frac{(14 + 15)}{35}$$

$$= \frac{29}{35}$$

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Addition of Mixed Numbers

When mixed numbers are to be added together, the whole numbers and the fractions are added separately.

Example:

Add $4\frac{2}{3}$ and $2\frac{3}{5}$

$$\begin{aligned} 4\frac{2}{3} + 2\frac{3}{5} &= 6 + \frac{2}{3} + \frac{3}{5} \\ &= 6 + \frac{10}{15} + \frac{9}{15} \\ &= 6 + \frac{19}{15} \\ &= 6 + 1 + \frac{4}{15} \\ &= 7\frac{4}{15} \end{aligned}$$

Subtraction of Mixed Numbers

To subtract two fractions which do not do have the same denominator, a method similar to that for addition is used.

Example:

Subtract $\frac{3}{4}$ from $\frac{5}{6}$

The lowest common denominator is 12.

$$\begin{aligned} \frac{5}{6} - \frac{3}{4} &= \frac{10}{12} - \frac{9}{12} \\ &= \frac{(10 - 9)}{12} \\ &= \frac{1}{12} \end{aligned}$$

When mixed numbers are involved first subtract the whole numbers and then deal with the fractional parts.

Example:

Subtract $4\frac{1}{3}$ from $6\frac{3}{4}$

$$\begin{aligned} 6\frac{3}{4} - 4\frac{1}{3} &= 2 + \frac{3}{4} - \frac{1}{3} \\ &= 2 + \frac{(9 - 4)}{12} \\ &= 2 + \frac{5}{12} \\ &= 2\frac{5}{12} \end{aligned}$$

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Multiplication of Fractions

Multiplication of fractions is performed by multiplying the numerators of each fraction to form the product numerators, and multiplying the individual denominators to form the product denominator. The resulting fraction is then reduced to its lowest terms.

Example:

Multiply $\frac{3}{8} by \frac{5}{7}$

$$\frac{3}{8} \times \frac{5}{7} = \frac{(3 \times 5)}{(8 \times 7)}$$

$$= \frac{15}{56}$$

If any factors are common to a numerator and a denominator they should be cancelled before multiplying.

Example:

Find the value of $\frac{2}{3} \times \frac{5}{7} \times \frac{21}{32}$

$$\frac{2}{3} \times \frac{5}{7} \times \frac{21}{32} = \frac{(1 \times 5 \times 1)}{(1 \times 1 \times 16)}$$

$$= \frac{5}{16}$$

Mixed numbers must be converted into improper fractions before multiplying.

Example:

Multiply $1\frac{3}{8} \times 2\frac{1}{3}$

$$1\frac{3}{8} \times 2\frac{1}{3} = \frac{11}{8} \times \frac{7}{3}$$

$$= \frac{(11 \times 7)}{(8 \times 3)}$$

$$= \frac{77}{24}$$

$$= 3\frac{5}{24}$$

In problems with fractions the word “of” is frequently used. It should always be taken as meaning “multiply”.

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Division of Fractions

Division of common fractions is accomplished by inverting, or turning over, the divisor and then multiplying. However, it is important that you invert the divisor only and not the dividend. Once the divisor is inverted, multiply the numerators to obtain a new numerator, multiply the denominators to obtain a new denominator. Then reduce the quotient to its lowest terms.

Example:

Divide $\frac{3}{5} by \frac{7}{8}$

$$\frac{3}{5} \div \frac{7}{8} = \frac{3}{5} \times \frac{8}{7}$$

$$= \frac{(3 \times 8)}{(5 \times 7)}$$

$$= \frac{24}{35}$$

Mixed numbers must be converted into improper fractions before dividing.

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Decimals

General

Working with fractions is typically time consuming and complex. One way you can eliminate fractions in complex equations is by replacing them with **decimal fractions** or decimals. A common fraction is converted to a decimal fraction by dividing the numerator by the denominator.

For example, ¾ is converted to a decimal by dividing the 3 by the 4.

The decimal equivalent of ¾ is 0.75. Improper fractions are converted to decimals in the same manner. However, whole numbers appear to the left of the decimal point.

In a decimal, each digit represents a multiple of ten. The first digit represents tenths, the second hundredths, the third thousandths.

Examples:

0.5 is read as five tenths

0.05 is read as five hundredths

0.005 is read as five thousandths

When writing decimals, the number of zeros to the right of the decimal does not affect the value as long as no other number except zero appears. In other words, numerically, 2.5, 2.50 and 2.500 are the same.

Decimal Places

The number of digits after the decimal point is called decimal places.

Examples:

27.6	one decimal place
27.16	two decimal places
27.026	three decimal places and so on.

Adding Decimals

The addition of decimals is done in the same manner as the addition of whole numbers. However, care must be taken to correctly align the decimal points vertically.

Example:

Add the following 25.78 + 5.4 + 0.237

Rewrite with the decimals aligned and add.

```
 25.78
+ 5.4
+ 0.237
 31.417
```

Once everything is added, the decimal point in the answer is placed directly below the other decimal points.

Subtracting Decimals

Like adding, subtracting decimals is done in the same manner as with whole numbers. Again, it is important that you keep the decimal points aligned.

Example:

If you have 325.25 pounds of ballast on board and remove 30.75 pounds, how much ballast remains?

```
 325.25
– 30.75
 294.50
```

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Multiplying Decimals

When multiplying decimals, ignore the decimal points and multiply the resulting whole numbers.

Once the product is calculated, count the number of digits to the right of the decimal point in both the multiplier and multiplicand. This number represents how many digits must be right of the decimal point in the product.

Example:

Multiplicand is 26.757 (3 decimal places)

Multiplier is 0.32 (2 decimal places)

Write:

26.757 x 0.32

Calculation:

26 757 x 32
80271
+ 53514
856224

The product is 856224.

Count 5 decimal places to the left of the last digit. Then set the decimal point.

The result is 8.56224.

Dividing Decimals

When dividing decimals, the operation is carried out in the same manner as division of whole numbers. However, to ensure accurate placement of decimal point in the quotient, two rules apply:

1. When the divisor is a whole number, the decimal point in the quotient aligns vertically with the decimal in the dividend when doing long division.
2. When the divisor is a decimal fraction, it should first be converted to a whole number by moving the decimal point to the right. However, when the decimal in the divisor is moved, the decimal in the dividend must also move in the same direction and the same number of spaces.

Example:

Divide 37.26 by 2.7.

Dividend is 37.26

Divisor is 2.7

Move the decimal in the divisor to the right to convert it to a whole number.

Move the decimal in the dividend the same number of places to the right.

Calculation:

372.6 : 27 = 13.8

27
102
81
216
216
0

1. **37** ÷ **27** = **1** Remainder 10
2. **102** ÷ **27** = **3** Remainder 21
3. Take 6 and set the point
4. **216** ÷ **27** = 8 Remainder 0

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Ratio

General

A ratio is a comparison between two similar quantities.

If the length of an aircraft is 72 m and a model of it is 1 m long then the length of the model is $\frac{1}{72}$ of the length of the aircraft. In making the model, all the dimensions of the aircraft are reduced in the ratio of 1 to 72.

The ratio 1 to 72 is usually written 1 : 72.

This ratio can also be written as the fraction $\frac{1}{72}$.

Before we can state a ratio the units must be the same. We can state a ratio between 3 mm and 2 m only if we bring both lengths to the same units. If we convert 2 m to 2000 mm the ratio between both lengths is 3:2000.

Examples

Express the following ratios as fractions reduced to their lowest terms.

Example 1:

40 mm to 2.2 m

2.2 m = 2200 mm

40 : 2200 = $\frac{40}{2200} = \frac{1}{55}$

Example 2:

800 g to 1.6 kg

1.6 kg = 1600 g

800 : 1600 = $\frac{800}{1600} = \frac{1}{2}$

Engine/Propeller Ratio

The use of ratios is common in aviation.

For example, if an engine turns at 4000 revolutions per minute or rpm in short, and the propeller turns at 2400 rpm, the ratio of the two speeds is 4000 to 2400, or 5 to 3, when reduced to lowest terms. This relationship can also be expressed as 5/3 or 5:3.

Compression Ratio

One ratio you must be familiar with is compression ratio, which is the ratio of cylinder displacement when the piston is at bottom centre to the cylinder displacement when the piston is at top centre.

For example, if the volume of a cylinder with the piston at bottom centre is 400 cubic centimetres and the volume with the piston at top centre is 50 cubic centimetres, the compression ratio is 400 : 50 or **8 :1** when simplified.

Gear Ratio

Another typical ratio is that of different gear sizes.

For example, the gear ratio of a drive gear with 15 teeth to a driven gear with 45 teeth is 15:45 or **1:3** when reduced.

This means that for every one tooth on the drive gear there are three teeth on the driven gear.

However, when working with gears, the ratio of teeth is opposite the ratio of revolutions. In other words, since the drive gear has one third as many teeth as the driven gear, the drive gear must complete three revolutions to turn the driven gear one revolution. This results in a revolution ratio of 3:1, which is opposite the ratio of teeth.

When d is the diameter of the rotating gears, the following ratio is true:

$$i = \frac{n_1}{n_2} = \frac{d_2}{d_1}$$

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Proportion

General

A proportion is a statement of equality between two or more ratios and represents a convenient way to solve problems involving ratios.

For example, if an engine has a reduction gear ratio between the crankshaft and the propeller of 3:2 and the engine is turning with n_1 = 2700 rpm, what is the speed n_2 of the propeller?

In this problem, let n_2 represent the unknown value, which in this case is the speed of the propeller. Next set up a proportional statement using the fractional form $\frac{3}{2} = \frac{2700}{n_2}$.

To solve this equation, cross multiply to arrive at the equation

$3 \times n_2 = 2 \times 2700$

To solve for n_2, divide 5400 by 3. The speed of the propeller is 1800 rpm.

$$\frac{3}{2} = \frac{(engine-speed)}{(propeller-speed)}$$

$$\frac{3}{2} = \frac{2700}{n_2}$$

$$3\,n_2 = 2700\ rpm \times 2 = 5400\ rpm$$

$$n_2 = 1800\ rpm$$

This same proportion may also be expressed as **3 : 2 = 2700 : n_2**.

The first and last terms of the proportion are called **extremes**, and the second and third terms are called the **means**. In any proportion, the product of the extremes is equal to the product of the means.

In this example, multiply the extremes to get $3 \times n_2$, and multiply the means to get 2 x 2700 or 5400. This results in the identical derived earlier;
$3 \times n_2 = 5400$.

3 : 2 = engine speed : propeller speed

3 : 2 = 2700 : n_2

3 · n_2 = 2 · 2700

3 · n_2 = 5400

n_2 = 1800 rpm.

Direct Proportion

If 5 litres of oil has a mass of 4 kg, then 10 litres of the same oil will have a mass of 8 kg. That is, if we double the quantity of oil its mass is also doubled.

This is an example of direct proportion. As the quantity of oil increases the mass increases in the same proportion. As the quantity of oil decreases the mass decreases in the same proportion.

Electrical Resistance

The electrical resistance of a wire 150 mm long is 2 ohms (Ω). Find the resistance of a similar wire which is 1 m long!

The lengths of the two wires are increased in the ratio of 1000 :150. The resistance will also increase in the ratio 1000 :150.

Thus resistance of a wire with 1m length = $2\,\Omega \times \frac{1000}{150}$ = <u>13.3 Ω</u>

Inverse Proportion

A motor car will travel 30 km in 1 hour if its speed is 30 km per hour.

If its speed is increased to 60 km per hour the time taken to travel 30 km will be ½ hour. That is when the speed is doubled the time taken is halved. This is an example of inverse proportion. When we multiply the speed by 2 we divide the time taken by 2.

Pulleys

Two pulleys of 150 mm and 50 mm diameter are connected by a belt. If the larger pulley revolves at 80 rpm find the speed of the smaller pulley!

The smaller pulley must revolve faster than the larger pulley and hence the quantities, speed and diameter, are in inverse proportion. The pulley diameters are decreased in the 50 : 150, or 1 : 3. The speed will be increased in the ratio of 3 :1.

Therefore the speed of smaller pulley = $80 \times \frac{3}{1}$ = <u>240 rpm</u>.

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Proportional Parts

The diagram below shows the line $\overline{AB}$ whose length represents 10 m divided into two parts in the ratio 2:3.

From the diagram the line has been divided into a total of 5 parts. The length $\overline{AC}$ contains 2 parts and the length $\overline{BC}$ contains 3 parts. Each part is 2 m long, hence $\overline{AC}$ is 4 m long and $\overline{BC}$ is 6 m long.

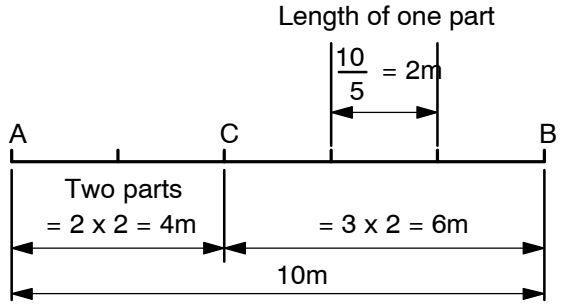


Figure 3 Proportional Parts

We could tackle the problem as follows;

Total number of parts = 2 + 3 = 5

Length of each part = $\frac{10}{5}$ = 2 m

Length of AC = 2 x 2 = 4 m

Length of BC = 3 x 2 = 6 m

Example:

A certain brass is made by alloying copper and zinc in the ratio of 7:3. How much copper must be mixed with 30 g of zinc?

3 parts have a mass of 30 g

1 part has a mass of 10 g

7 parts have a mass of 70 g

Therefore, mass of copper needed = 70 g.

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Percentages

General

The wording comes from the Latin expression “per centum“. That means “per hundred“.

“Cent“ is also the French word for “hundred“.

Concerning money, hundred cents make one Euro.
In the USA hundred cents make one dollar.

Mathematics

In mathematics this term expresses “how much of the whole thing“.

The “whole thing“ may be divided in one hundred parts of equal size.

If you take three of these parts, you have three percent.

Symbol

The sign for “percent“ is %.

Three percent is written 3%.

Calculation

The percentage of an amount can be calculated by the use of multiplication and division.

The percentage is calculated by multiplying the actual value by 100 and then dividing the result by the maximum value.

$$x\% = \frac{(actual\ \ value \times 100)}{\{maximum\ \ value\}}$$

Example:

The fuel tank of a car has a capacity of 60 litres.

The actual value is 24 litres.

The actual value is how many percent of the maximum value?

Calculation:

The calculation is 24 litres multiplied by 100. The result is divided by 60 litres.

$$x\% = \frac{(24l \times 100)}{\{60l\ \}} = 40\%$$

The result is 40%.

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Averages

General

An average is also called arithmetic mean and is the mean value that is defined by the sum of all values divided by the quantities of the values. Usually it is denoted by this symbol: ∅

Calculation

The average of an amount can be calculated by the use of addition and division.

The average is calculated by adding the values and then dividing the result by the number of values.

$$x = \frac{(value1 + value2 + value3 + value\ n)}{number\ of\ values}$$

Example:

In a small village, let us call it Smalltown, there are ten families. You want to know the average monthly income before tax and other dues.

Family Smith, family Miller and family Jones have 2 500 Euros each.

Family Johnson, family Carter and family Adams have 3 500 Euros.

Family Jackson and family Jefferson have 4 000 Euros.

Family Washington lives from social welfare and has 1 000 Euros.

Family Madison is very healthy and has an income of 20 000 Euros.

$$x = \frac{[(3 \times 2500) + (3 \times 3500) + (2 \times 4000) + 1000 + 20000]}{10} = \frac{47000}{10}$$

village ∅ = 4700 Euros.

The calculation is adding all monthly incomes. This will be 47 000 Euros in the complete village.

Divided by the number of families which is ten, the average income in Smalltown is 4 700 Euros.

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Power and Roots

Base and Exponent

When a number is multiplied by itself, it is said to be raised to a given power. For example, 6 x 6 = 36; therefore, 6^2 = 36.

The number of times a **base number** is multiplied by itself is expressed as an **exponent**. It is written to right slightly above the base number.

The quantity $2 \times 2 \times 2 \times 2$ may be written as 2^4.

Now 2^4 is called the fourth power of the base 2.

The number 4, which gives the number of 2s to be multiplied together is the exponent.

Similarly $a \times a \times a = a^3$

Here a^3 is the third power of the base a, and the index is 3.

Thus in the expression x^n

x^n is called the nth power of x
x is called the base, and
n is called the exponent.

Remember that, in algebra, letters such as a in the above expression merely represent numbers.

Index

A different name for the exponent is “index“.

Exponent Zero

Any number, except zero, that is raised to the zero power equals 1.

Example:

$14^0 = 1$

Positive Exponent

A positive exponent indicates how many times a number is multiplied by itself.

When the exponent does not have a sign (+ or –) preceding it, the exponent is assumed to be positive.

Squaring

3^2 is read 3 squared or 3 to the second power.

Its value is found by multiplying 3 by itself 2 times.

3 x 3 = 9

Cubing

2^3 is read 2 cubed or 2 to the third power.

Its value is found by multiplying 2 by itself 3 times.

2 x 2 x 2 = 8

Negative Exponent

A negative exponent implies the division of 1 by the value which is given by a base number with its exponent.

It indicates the inverse, or reciprocal of the number with its exponent made positive.

Example:

2^{-3} is read 2 to the negative third power.

The inverse, or reciprocal of 2^{-3} with its exponent made positive is

$$\frac{1}{2^3} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Reciprocals

Hence the laws of arithmetic apply strictly to algebraic terms as well as numbers. The expression $\frac{1}{2}$ is called the reciprocal of 2.

Similarly the expression $\frac{1}{p}$ is called the reciprocal of p likewise the expression $\frac{1}{x^n}$ is called the reciprocal of x^n.

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Roots

General

The root of a number is that value which, when multiplied by itself a certain number of times, produces that number.

For example, 4 is a root of 16 because when multiplied by itself, the product is 16.

However, 4 is also a root of 64 because 4 x 4 x 4 = 64.

Symbol

The symbol used to indicate a root is the radical sign ($\sqrt{x}$) placed over the number.

Square Root

If only the radical sign appears over a number, it indicates you are to extract the square root of the number under the sign.

For example the square root is expressed as $\sqrt{16}$.

Index Number

Is the root of a number other than a square root, an index number is placed outside the radical sign.

For example the cube root is expressed as $\sqrt[3]{64}$.

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Laws of Indices

Multiplication

Let us see what happens when we multiply powers of the same base together.

$$5^2 \times 5^4 = (5 \times 5) \times (5 \times 5 \times 5 \times 5)$$
$$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$$

or

$$c^3 \times c^5 = (c \times c \times c) \times (c \times c \times c \times c \times c)$$
$$= c \times c \times c \times c \times c \times c \times c \times c = c^8$$

In both the examples above we see that we could have obtained the result by adding the indices together.

$$5^2 \times 5^4 = 5^{2+4} = 5^6$$

$$c^3 \times c^5 = c^{3+5} = c^8$$

We may apply this idea when multiplying more than two powers of the same base together.

Thus $7^2 \times 7^5 \times 7^9 = 7^{2+5+9} = 7^{16}$

The law is:

When multiplying powers of the same base together, add the indices.

Division of Powers

Now let us see what happens when we divide powers of the same base.

$$\frac{3^5}{3^2} = \frac{(3 \times 3 \times 3 \times 3 \times 3)}{(3 \times 3)} = 3 \times 3 \times 3 = 3^3$$

We see that the same result could have been obtained by subtracting the indices.

$$\frac{3^5}{3^2} = 3^{5-2} = 3^3$$

The law is:

When dividing powers of the same base subtract the index of the denominator from the index of the numerator.

Powers of Powers

How do we simplify $(5^3)^2$?

One way is to proceed as follows:

$$(5^3)^2 = 5^3 \cdot 5^3 = 5^{3+3} = 5^6$$

We see that the same result would have been obtained if we multiply the two indices together.

$$(5^3)^2 = 5^{3x2} = 5^6$$

The law is:

When raising the power of a base to a power, multiply the indices together.

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Zero Index

Now $\frac{2^5}{2^5} = \frac{(2 \times 2 \times 2 \times 2 \times 2)}{(2 \times 2 \times 2 \times 2 \times 2)} = 1$

But using the laws of indices

$$\frac{2^5}{2^5} = 2^{5-5} = 2^0 \text{ Thus} \qquad 2^0 = 1$$

Also $\frac{c^4}{c^4} = \frac{(c \times c \times c \times c)}{(c \times c \times c \times c)} = 1$ or $\frac{c^4}{c^4} = c^{4-4} = c^0 = 1$

The law is:

Any base raised to the index zero is equal to 1.

Negative Indices

Now $\frac{2^3}{2^7} = \frac{(2 \times 2 \times 2)}{(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)} = \frac{1}{(2 \times 2 \times 2 \times 2)} = \frac{1}{2^4}$ but using the laws of indices $\frac{2^3}{2^7} = 2^{3-7} = 2^{-4}$

it follows that $2^{-4} = \frac{1}{2^4}$

The law is:

The power of a base which has a negative index is the reciprocal of the power of the base having the same, but positive, index.

Negative Indices are often used to express prefixes in the metric systems, for example millimetres.

If an object is only 0.0025 metres long, we can write $2.5 \times 10^{-3}\ metres$.

In physics, the basis 10 is very often used with the index -3, -6 or -9.

Fractional Indices

The cube root of 5 (written as $\sqrt[3]{5}$) is the number which, when multiplied by itself three times, gives 5.

$$\sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5} = 5$$

but we also know that $5^{1/3} \times 5^{1/3} \times 5^{1/3} = 5^{1/3+1/3+1/3} = 5$

Comparing these expressions

$$\sqrt[3]{5} = 5^{1/3}$$

Similarly the fourth root of base d (written as $\sqrt[4]{d}$) is the number which, when multiplied by itself four times, gives d.

$$\sqrt[4]{d} \times \sqrt[4]{d} \times \sqrt[4]{d} \times \sqrt[4]{d} = d$$

But we also know that $d^{1/4} \times d^{1/4} \times d^{1/4} \times d^{1/4} = d^{1/4+1/4+1/4+1/4} = d$

Comparing these expressions $\sqrt[4]{d} = d^{1/4}$

The law is:

A fractional index represents a root, the denominator of the index denotes the root to be taken.

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Areas

General

The area of a plane figure is measured by seeing how many square units it contains.

1 square metre is the area contained in a square having a side of 1 metre; 1 square centimetre is the area contained in a square having a side of 1 centimetre, etc.

The standard abbreviations are:

1 square metre	1 m^2
1 square centimetre	1 cm^2
1 square millimetre	1 mm^2
1 square inch	1 in^2
1 square foot	1 ft^2
1 square yard	1 yd^2

The following provides the formulae for areas and perimeters of simple geometrical shapes.

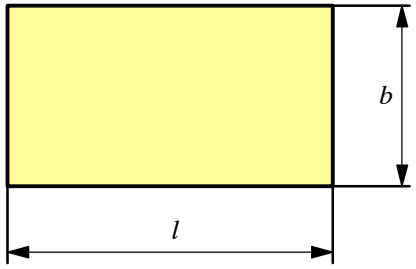


Figure 4 Rectangle

Area = $l \times b$

Perimeter = $2l + 2b$

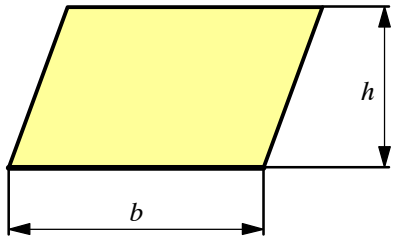


Figure 5 Parallelogram

Area = $b \times h$

Perimeter = Sum of all 4 sides.

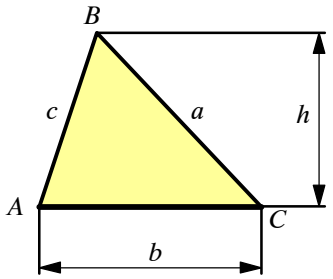


Figure 6 Triangle

Area = $\sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{(a+b+c)}{2}$

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Area of a Triangle

General

To calculate the area of a triangle, we can use the following formula:

Area = ½ × base × height

Any side can be chosen as base providing you use the corresponding height.

Example 1:

In our first example, we have the height = 12 cm and the base = 10 cm

∴ Area = ½ x base x height

= ½ × 10cm × 12cm

= 60 cm^2

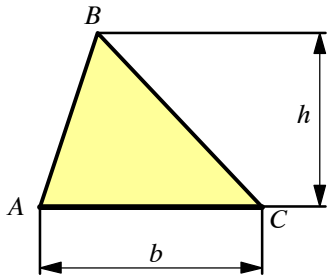


Figure 7 Area of Triangle

Area of a Right–Angled Triangle

The height of a right–angled triangle is just one of its sides.

Example:

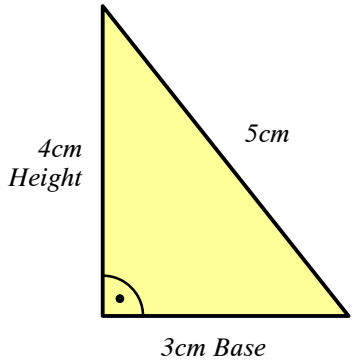


Figure 8 Area of Right-Angled Triangle

Area = ½ × base × height

= ½ × 3 × 4

= 6 cm^2

Obviously, we could tip the triangle on its side.

The 4 cm side now becomes the base and the 3 cm side becomes the height. Note, no difference in size of area.

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Area of Circle

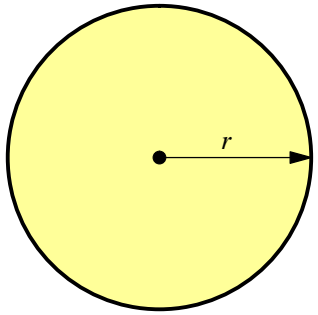


Figure 9 Circle

Area = πr^2

Circumference = $2 \pi r = \pi d$

$\pi = 3.142$ or $\frac{22}{7}$

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Volumes

General

The volume of a solid figure is found by seeing how many cubic units it contains. 1 cubic metre is the volume contained inside a cube having an edge 1 metre long; 1 cubic centimetre is the volume contained inside a cube having an edge 1 centimetre long, etc.

The standard abbreviations for units of volume are as follows:

1 cubic metre	1 m^3
1 cubic centimetre	1 cm^3
1 cubic millimetre	1 mm^3
1 cubic inch	1 in^3
1 cubic foot	1 ft^3
1 cubic yard	1 yd^3

The following figures give the formulae for the volumes and surface areas of solid figures.

For any solid having a uniform cross–section there are formulae to calculate volume and surface area.

Volume

Volume = Cross–sectional area x Length of solid

Surface Area

Surface Area = Lateral Surface + Ends
i.e. (perimeter of cross–section × Length of Solid) + (Total area of ends)

Cuboid

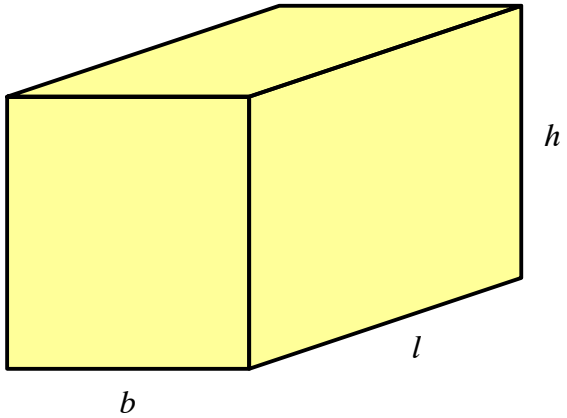


Figure 10 Cuboid

Volume = l × b × h

Surface Area = 2 (l × b + l × h + b × h)

Cylinder

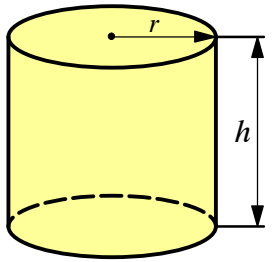


Figure 11 Cylinder

Volume = $\pi r^2 \times h$

Surface Area = $2 \pi r (h + r)$

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Sphere

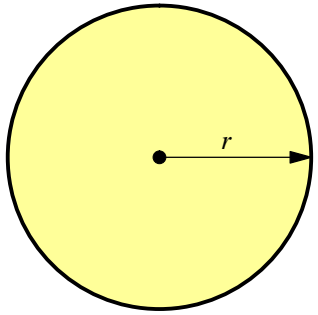


Figure 12 Sphere

Volume = $\frac{4}{3}\pi r^3$

Surface Area = $4\pi r^2$

Pyramid

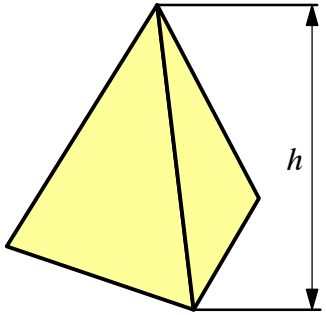


Figure 13 Pyramid

Volume = $\frac{1}{3}Ah$

Surface Area = Sum of the areas of the triangles forming the sides plus the area of the base

(A = Area of base)

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M1.2 ALGEBRA

Introduction

The methods of algebra are an extension of those used in arithmetic. In algebra we use letters and symbols as well as numbers to represent quantities.

When we write that a sum of money is £ 50 we are making a particular statement but if we write a sum of money is £ *P* we are making a general statement. This general statement will cover any number we care to substitute for *P*.

Use of Symbols

A technician often has to indicate that certain quantities or measurements have to be added, subtracted, multiplied or divided. Frequently this has to be done without using actual numbers.

The statement:

Area of a rectangle = length x breadth

is a perfectly general statement which applies to all rectangles. If we use symbols we obtain a much shorter statement.

if A = the area of the rectangle

l = the length of the rectangle

and b = the breadth of the rectangle

then the statement becomes:

$A = l \times b$

Knowing what the symbols A, l and b stand for, this statement conveys as much information as the first statement. To find the area of a particular rectangle we replace the symbols l and b by the actual dimensions of the rectangle, first making sure that l and b have the same units. To find the area of a rectangle whose length is 50mm and whose breadth is 30 mm we put l = 50 mm and b = 30 mm.

$A = l \times b = 50 \times 30 = 1500\ mm^2$

Many verbal statements can be translated into symbols as the following statements show:

The difference of two numbers $= x - y$

Two numbers multiplied together $= a \times b$

One number divided by another $= p \div q$

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Substitution

General

The process of finding the numerical value of an algebraic expression for given values of the symbols that appear in it is called substitution.

Example:

If x = 3, y = 4 and z = 5 find the value of:

$$\frac{(3y + 2z)}{(x + z)} = \frac{(3 \times 4) + (2 \times 5)}{3 + 5}$$

$$\frac{(12 + 10)}{8} = \frac{22}{8} = 2.75 \text{ or } 2\tfrac{3}{4}.$$

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Addition & Subtraction of Algebraic Terms

General

Like terms are numerical multiples of the same algebraic quantity.

7x, 5x and –3x

are three like terms.

An expression consisting of like terms can be reduced to a single term by adding or subtracting the numerical coefficients.

$7x - 5x + 3x = (7 - 5 + 3)\,x = 5x$

$3b^2 + 7b^2 = (3 + 7)b^2 = 10b^2$

$-3y - 5y = (-3 - 5)\,y = -8y$

$q - 3q = (1 - 3)\,q = -2q$

Only like terms can be added or subtracted. Thus 7a + 3b – 2c is an expression containing three unlike terms and it cannot be simplified any further.

Similarly with $8a^2b + 7ab^3 - 6a^2b^2$ which are all unlike terms. It is possible to have several sets of like terms in an expression and each set can then be simplified.

$8x + 3y - 4z - 5x + 7z - 2y + 2z$

$= (8 - 5)x + (3 - 2)y + (-4 + 7 + 2)z$

$= 3x + y + 5z$

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Multiplication & Division Signs

General

When using letters multiplication signs are nearly always omitted and l x b becomes lb.

Of course the same scheme cannot apply to numbers and we cannot write 9 x 6 as 96.

The multiplication sign can, however, be omitted when a symbol and a number are to be multiplied together. Thus 5 x m is written 5m.

The system may be extended to three or more quantities and hence *P x L x A x N* is written *PLAN.* The symbols need not be written in any special order because the order in which numbers are multiplied together is unimportant. Thus *PLAN* is the same as *LANP* or *NAPL.*

It is usual, however, to write numbers before letters and to arrange letters in alphabetical order, that is, it is better to write 8xy than xy8 or x8y. In algebraic expressions the number in front of the symbols is called the coefficient. Thus in the expression 8x the coefficient of x is 8.

Division Sign

The division sign ÷ (often :) is seldom used in algebra and it is more convenient to write P ÷ q in the fractional form $\frac{p}{q}$

Example:

$$\frac{lp}{2\pi R} = lp \div 2\pi R$$

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Multiplication & Division of Algebraic Quantities

General

The rules are exactly the same as those used with directed numbers.

$(+x)(+y) = +(xy) = +xy = xy$

$5x \times 3y = 5 \times 3 \times x \times y = 15xy$

$(x)(-y) = -(xy) = -xy$

$(2x)(-3y) = -(2x)(3y) = -6xy$

$(-4x)(2y) = -(4x)(2y) = -8xy$

$(-3x)(-2y) = +(3x)(2y) = 6xy$

$\frac{(+x)}{(+y)} = +\frac{x}{y} = \frac{x}{y}$

$\frac{(-3x)}{2y} = -\frac{3x}{2y}$

$\frac{(-5x)}{(-6y)} = +\frac{5x}{6y} = \frac{5x}{6y}$

$\frac{4x}{(-3y)} = -\frac{4x}{3y}$

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Simplification of Terms

Powers

When multiplying expressions containing the same symbols, indices are used:

$m \times m = m^2$

$3m \times 5m = 3 \times m \times 5 \times m = 15m^2$

$(-m) \times m^2 = (-m) \times m \times m = -m^3$

$5m^2n \times 3mn^3 = 5 \times m \times m \times n \times 3 \times m \times n \times n \times n = 15m^3n^4$

$3mn \times (-2n^2) = 3 \times m \times n \times (-2) \times n \times n = -6mn^3$

Fractions

When dividing algebraic expressions, cancellation between numerator and denominator is often possible, cancelling is equivalent to dividing both numerator and denominator by the same quantity:

$$p\,\frac{q}{p} = \frac{(p \times q)}{p} = q$$

$$\frac{(3\,p^2\,q)}{(6\,p\,q^2)} = \frac{(3 \times p \times p \times q)}{(6 \times p \times q \times q)} = \frac{3p}{6q} = \frac{p}{2q}$$

$$\frac{(18\,x^2\,y^2\,z)}{(6\,x\,y\,z)} = \frac{(18 \times x \times x \times y \times y \times z)}{(6 \times x \times y \times z)} = 3\,x\,y$$

Sequence

Remember the word BODMAS which gives the initial letters of the correct sequence i.e. Brackets, Of, Division, Multiply, Add, Subtract.

Thus

$$\begin{aligned} 2x^2 + (12x^4 - 3x^4) \div 3x^2 - x^2 &= 2x^2 + 9x^4 \div 3x^2 - x^2 \\ &= 2x^2 + 3x^2 - x^2 \\ &= 5x^2 - x^2 \\ &= 4x^2 \end{aligned}$$

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Indices

Negative Indices

The indices of powers are not always “normal“ positive numbers.

Negative indices may also be used. By this, a fraction line can be avoided and the term is clearly laid out.

A power with negative index is written like this: a^{-b}

This is the same as $\frac{1}{a^b}$.

Fractional Indices

Fractional indices of power may also be used. By this, the sign for a root can be avoided and the term is clearly laid out.

A fractional index is written like this: $a^{\frac{b}{c}}$

This is the same as $\sqrt[c]{a^b}$

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Brackets

General

Brackets are used for convenience in grouping terms together. When removing brackets each term within the bracket is multiplied by the quantity outside the bracket:

$$3(x+y) = 3x+3y$$

$$5(2x+3y) = 5 \times 2x+5 \times 3y = 10x+15y$$

$$4(a-2b) = 4 \times a-4 \times 2b = 4a-8b$$

$$m(a+b) = ma+mb$$

$$3x(2p+3q) = 3x \times 2p+3x \times 3q = 6px+9qx$$

$$4a(2a+b) = 4a \times 2a+4a \times b = 8a^2+4ab$$

Simplifying

When simplifying expressions containing brackets first remove the brackets and then add the like terms together.

$$(3x+7y)-(4x+3y) = 3x+7y-4x-3y = -x+4y$$

$$3(2x+3y)-(x+5y) = 6x+9y-x-5y = 5x+4y$$

$$x(a+b)-x(a+3b) = ax+bx-ax-3bx = -2bx$$

$$2(5a+3b)+3(a-2b) = 10a+6b+3a-6b = 13a$$

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Factorization

General

A factor is a common part of two or more terms which make up an algebraic expression.

Thus the expression $3x+3y$ has two terms which have the number 3 common to both of them. Thus $3x+3y = 3(x+y)$. We say that 3 and $(x+y)$ are the factors of $3x+3y$.

Highest Common Factor

To factorize algebraic expressions of this kind, we first find the Highest Common Factor (HCF) of all the terms making up the expression. The HCF then appears outside the bracket. To find the terms inside the bracket divide each of the terms making up the expression by the HCF.

Example 1:

Find the factors of $ax+bx$

The HCF of ax and bx is x

$$ax+bx = x(a+b)$$

Example 2:

Find the factors of m^2n-2mn^2

The HCF of m^2n and $2mn^2$ is mn

$$m^2n-2mn^2 = mn(m-2n)$$

Example 3:

Find the factors of $3x^4y+9x^3y^2-6x^2y^3$

The HCF of $3x^4y$, $9x^3y^2$ and $6x^2y^3$ is $3x^2y$

$$3x^4y+9x^3y^2-6x^2y^3 = 3x^2y(x^2+3xy-2y^2)$$

Example 4:

Find the factors of $\frac{ac}{x}+\frac{bc}{x^2}-\frac{cd}{x^3}$

The HCF of $\frac{ac}{x}$, $\frac{bc}{x^2}$ and $\frac{cd}{x^3}$ is $\frac{c}{x}$

$$\frac{ac}{x}+\frac{bc}{x^2}-\frac{cd}{x^3} = \frac{c}{x}\left(a+\frac{b}{x}-\frac{d}{x^2}\right)$$

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Factorizing by Grouping

To factorize the expression ax + ay + bx + by first group the terms in pairs so that each pair of terms has a common factor.

Example 1:

Factorize ax + ay + bx + by

$$ax + ay + bx + by = (ax + ay) + (bx + by)$$
$$= a(x + y) + b(x + y)$$
$$= (a + b)\,(x + y)$$

Example 2:

Factorize mp + np − mq − nq

$$mp + np - mq - nq = (mp + np) - (mq + nq)$$
$$= p(m + n) - q(m + n)$$
$$= (p - q)\,(m + n)$$

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Factors of Quadratic Expressions

An expression of the type $ax^2 + bx + c$ is called a quadratic expression.

$x^2 - 5x + 1$ and $3p^2 + 12p - 5$

are both quadratic expressions.

Example 1:

In an expression like $x^2 + 7x + 12$, the factors will be of the type $(x + m)(x + n)$ where m and n are two numbers whose sum is 7 and whose product is 12. These two numbers must be 3 and 4.

So

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Example 2:

This technique is also useful when factorizing expressions such as $12x^2 + 23x + 10$ in which

$a = 12$, $b = 23$ and $c = 10$.

We can factorize if we can get two integers whose product is ac and whose sum is b.

Thus:

$$ac = 12 \times 10 = 120$$

$$b = 23.$$

Now $120 = 8 \times 15$. It can now be seen that the two integers are 8 and 15.
Since $23x = 15x + 8x$

$$12x^2 + 23x + 10 = 12x^2 + 15x + 8x + 10 = 3x(4x + 5) + 2(4x + 5)$$

$$= (4x + 5)(3x + 2)$$

Example 3:

Factorize $8x^2 - 34x + 21$

Here $a = 8, b = -34$ and $c = 21$.

$ac = 168$ and $b = -34$

The two integers whose sum is –34 and whose product is 168 are –6 and –28.

$$8x^2 - 34x + 21 = 8x^2 - 6x - 28x + 21$$

$$= 2x(4x - 3) - 7(4x - 3)$$

$$= (4x - 3)(2x - 7)$$

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Algebraic Fractions

General

Since algebraic expressions contain symbols (or letters) which represent numbers all the rule of operations with numbers also apply to algebraic terms, including fractions.

Thus

$$\frac{1}{\left(\frac{1}{a}\right)} = 1 \div \frac{1}{a} = 1 \times \frac{a}{1} = \frac{(1 \times a)}{1} = a$$

and

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

and

$$\frac{(x+y)}{\frac{1}{(x-y)}} = (x+y) \div \frac{1}{(x-y)} = (x+y) \times \frac{(x-y)}{1} = (x+y)(x-y)$$

You should note in the last example how we put brackets round $x + y$ and $x - y$ to remind us that they must be treated as single expressions, otherwise we may have been tempted to handle the terms x and y on their own.

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Adding & Subtracting Algebraic Fractions

Consider the expression $\frac{a}{b} + \frac{c}{d}$ which is the addition of two fractional terms. These are called partial fractions.

If we wish to express the sum of these partial fractions as one single fraction then we proceed as follows (similar method used when adding or subtracting number fractions)

First find the lowest common denominator. This is the LCM of b and d which is bd. each fraction is then expressed with bd as the denominator.

Example:

$\frac{a}{b} = \frac{a \times d}{b \times d} = \frac{ad}{bd}$ and $\frac{c}{d} = \frac{c \times b}{d \times b} = \frac{cb}{bd}$

and adding these new fractions we have:

$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{cb}{bd} = \frac{ad + cb}{bd}$

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Multiplication & Division of Algebraic Fractions

As with ordinary arithmetic fractions, numerators can be multiplied together, as can denominators, in order to form a single fraction.

Example 1:

$$\frac{a}{b} \times \frac{c}{d} = \frac{(a \times c)}{(b \times d)}$$

Example 2:

$$\frac{3x}{2y} \times \frac{p}{4q} \times \frac{r^2}{s} = \frac{(3x \times p \times r^2)}{(2y \times 4q \times s)}$$

Cancelling

Factors which are common to both numerator and denominator may be cancelled.

it is important to realize that this cancelling means dividing the numerator and denominator by the same quantity.

Example 3:

$$\frac{8ab}{3mn} \times \frac{9mn^2}{4ab^2} = \frac{(8 \times a \times b \times 9 \times n \times n \times m)}{(3 \times m \times n \times 4 \times a \times b \times b)}$$

$$= \frac{6n}{b}$$

Example 4:

$$\frac{(5x^2y)}{8ab^3} \div \frac{10xy}{(4a^2b)} = \frac{(5x^2y)}{8ab^3} \times \frac{(4a^2b)}{10xy}$$

$$= \frac{(5 \times x \times x \times y \times 4a \times a \times b)}{(8 \times a \times b \times b \times b \times 10 \times x \times y)}$$

$$= \frac{ax}{4b^2}$$

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Transposition of Formulae

General

The formula $y = ax + b$ has y as its subject. By rearranging this formula we could make x the subject. We are then said to have transposed the formula to make x the subject.

The rules for transforming a formula are:

1. Remove square roots or other roots.
2. Get rid of fractions.
3. Clear brackets.
4. Collect together the terms containing the required subject.
5. Factorise if necessary.
6. Isolate the required subject.

These steps should be performed in the given order.

Example 1:

Transpose the formula F = ma to make a the subject.

Step 1. Divide both sides by m. then,

$$\frac{F}{m} = \frac{ma}{m}$$

or $\frac{F}{m} = a$ or $a = \frac{F}{m}$

Example 2:

Transpose $x = \frac{y}{b}$ to make b the subject

Step 1. Multiply both sides by b. then,

$$x \times b = \frac{y}{b} \times b$$

$bx = y$ or $y = bx$

Example 3:

Transpose the formula $V = \frac{2R}{(R - r)}$ to make R the subject.

Step 1. Since there are no roots, get rid of the fraction by multiplying both sides of the equation by $(R - r)$.

$$V(R - r) = 2R$$

Step 2. Clear the bracket.

$$VR - Vr = 2R$$

Step 3. Collect the terms containing R on the LHS.

$$VR - 2R = Vr$$

Step 4. Factorise the LHS.

$$R(V - 2) = Vr$$

Step 5. Isolate R by dividing both sides of the equation by $(V - 2)$.

$$R = \frac{Vr}{(V - 2)}$$

Although we used five steps to obtain the required subject, in very many cases far fewer steps are needed. Nevertheless, you should work through the steps in the order given.

Example 4:

Transpose $d = \sqrt{2hr}$ to make h the subject.

Step 1. Remove the square root by squaring both sides.

$$d^2 = 2hr$$

Step 2. Since there are no fractions or brackets and factorisation is not needed we can now isolate h by dividing both sides of the equation by $2r$.

$$\frac{d^2}{2r} = h \text{ or } h = \frac{d^2}{2r}$$

Since it is usual to position the subject on the LHS.

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Linear Equations

General

An arithmetical quantity has a definite value, such as 93, 3.73 or $\frac{3}{4}$. An algebraic quantity, however, given by algebraic expressions such as $x - (-3)$ or x^2, represents many amounts depending on the value given to x.

In algebra there are two methods we use to show that two quantities are equivalent to each other. One is called an *identity* and the other an *equation*.

Identities

A statement of the type $x^2 \equiv x{\cdot}x$ is called an identity.

The sign $\equiv$ means "is identical to". Any statement using this sign is true for all values of the variable, the variable in this case being x.

Thus when $x = 2$ we have $2^2 \equiv 2{\cdot}2$

and when $x = 3$ we have $3^2 \equiv 3{\cdot}3$ and so on.

Another type of identity involves units as, for example, the relationship between kilometres and metres. This may be stated as

x km $\equiv 1000x$ m

Thus 7 km $\equiv$ 7000 m

and 9 km $\equiv$ 9000 m and so on.

In practise the $\equiv$ sign is often replaced by the = (equals) sign and the above identities would be stated as

$x^2 = x{\cdot}x$

and x km = 1000x m

Equations of First Degree

Linear equations are called equations of the first degree. The exponent of the variable is 1.

Equations

A statement of the type $x - 3 = 5$ is called an equation.

This means that the quantity on the left–hand side of the equation is equal to the quantity on the right–hand side.

We can see that, unlike an identity, there is only one value of x that will satisfy the equation, or make the left–hand side equal to the right–hand side. The process of finding $x = 8$ is called solving the equation, and the value 8 is known as the *solution* or *root* of the equation.

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Solving Linear Equations

Linear equations contain only the first power of the unknown quantity.

$$7t - 5 = 4t + 7 \text{ and } \frac{5x}{3} = \frac{(2x + 5)}{2}$$

are both examples of linear equations.

In the process of solving an equation the appearances of the equation may be considerable altered but the values on both sides must remain the same. We must maintain this equality, and hence whatever we do to one side of the equation we must do exactly the same to the other side.

Check

After an equation is solved, the solution should be checked by substituting the result in each side of the equation separately. If each side of the equation then has the same value the solution is correct. In the detail which follows, LHS means left–hand side and RHS means right–hand side.

Equations Requiring Multiplication & Division

Example:

Solve the equation $\frac{x}{6} = 3$

Multiply each side by 6, we get

$$\frac{x}{6} \times 6 = 3 \times 6$$

$$x = 18$$

Check: when $x = 18$, LHS = $\frac{18}{6}$, RHS = 3

Equations Requiring Addition & Subtraction

Example:

Solve the equation $x - 4 = 8$

If we add 4 to each side, we get

$$x - 4 + 4 = 8 + 4$$

$$x = 12$$

The operation of adding 4 to each side is the same as transferring –4 to the RHS but in so doing the sign is changed from a minus to a plus.

$$x - 4 = 8$$

$$x = 8 + 4$$

$$x = 12$$

Check: when $x = 12$, LHS = 12 – 4 = 8, RHS = 8

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Equations Containing the Unknown Quantity on Both Sides

In equations of this kind, group all the terms containing the unknown quantity on one side of the equation and the remaining terms on the other side.

Example 1:

Solve the equation $7x + 3 = 5x + 17$

Subtracting $5x$ and 3 from both sides,

$$7x - 5x = 17 - 3$$
$$2x = 14$$
$$x = \frac{14}{2}$$
$$x = 7$$

Example 2:

Solve the equation $\frac{(4 - x)}{3} - \frac{(2x - 1)}{2} = 4$

In solving equations of this type remember that the line separating the numerator and denominator act as a bracket. The LCM of the denominators 3 and 2 is 6. Multiplying each term of the equation by gives:

$$\frac{(4 - x)}{3} \times 6 - \frac{(2x - 1)}{2} \times 6 = 4 \times 6$$
$$2(4 - x) - 3(2x - 1) = 24$$
$$8 - 2x - 6x + 3 = 24$$
$$- 8x + 11 = 24$$
$$- 8x = 24 - 11$$
$$- 8x = 13$$
$$x = \frac{13}{- 8}$$
$$x = - \frac{13}{8} = - 1.625$$

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Simultaneous Linear Equations

Consider the equations

$$3x + 5y = 21$$
$$2x + 3y = 13$$

each equation contains the unknown quantities x and y which satisfy both equations simultaneously. Equations like these are called simultaneous equations.

Solution of Simultaneous Linear Equations

The method for solving simultaneous equations is illustrated by the following examples.

Example 1:

Given:

$$5x + 3y = 19 \qquad (1)$$
$$3x + 2y = 12 \qquad (2)$$

If we multiply equation (1) by 3 and equation (2) by 5 the coefficient of x will be same in both equations.

$$15x + 9y = 57 \qquad (3)$$
$$15x + 10y = 60 \qquad (4)$$

We now eliminate x by subtracting equation (3) from equation (4) which gives

$$y = 3$$

To find the value of x we substitute for y in either of the original equations. Substituting $y = 3$, in equation (1)

$$5x + 3 \times 3 = 19$$
$$5x + 9 = 19$$
$$5x = 10$$
$$x = 2$$

Therefore the solutions are $y = 3$ and $x = 2$. To check these values substitute them in equation (2).

Example 2:

Given:

$$3x + 4y = 29 \qquad (1)$$
$$8x - 2y = 14 \qquad (2)$$

In these equations it is easier to eliminate y because the same coefficient of y can be obtained in both equations simply by multiplying equation (2) by 2.

$$16x - 4y = 28 \qquad (3)$$

Adding equations (1) and (3) gives

$$19x = 57$$
$$x = 3$$

Substituting $x = 3$ in equation (1) gives

$$3 \times 3 + 4y = 29$$
$$9 + 4y = 29$$
$$4y = 29 - 9$$
$$4y = 20$$
$$y = 5$$

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Quadratic Equations

General

An equation which can be written in the form $ax^2 + bx + c = 0$ is called a quadratic equation. The constants a, b and c can take any numerical value. The following are all examples of quadratic equations:

$$x^2 - 9 = 0$$

in which $a = 1, b = 0$ and $c = -9$

$$3x^2 + 8x + 5 = 0$$

in which $a = 3, b = 8$ and $c = 5$

$$4x^2 - 7x = 0$$

in which $a = 4, b = -7$ and $c = 0$

A quadratic equation has two solutions (often called the roots of the equation). It is possible for one of the roots to be zero or for the two solutions to be the same.

Equations of Second Degree

Quadratic equations are called equations of the second degree.

The exponent of the variables is never bigger than 2.

The variables do not have exponents like x^3 or x^4 .

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Solving Quadratic Equations

A quadratic equation can be solved by factorization. We make use of the fact that if the product of two factors is zero, then one of those factors must be zero.

Thus if $mn = 0$ then

- $m = 0$ or
- $n = 0$.

To solve a quadratic equation by this method the expression: $ax^2 + bx + c$ is written as the product of two factors.

Example 1:

Solve $x^2 + 7x + 12 = 0$

Factorizing, $(x + 4)(x + 3) = 0$

Either $x + 4 = 0$, giving $x = -4$ or $x + 3 = 0$, giving $x = -3$

The roots are $x = -4$ and $x = -3$.

Example 2:

Solve $x^2 - 6x = 0$

Factorizing $x(x - 6) = 0$

Either $x = 0$ or $x - 6 = 0$, giving $x = 6$.

The roots are $x = 0$ and $x = 6$

(Note that it is incorrect to say that the solution is $x = 6$. The solution $x = 0$ must also be stated).

Example 3:

Solve $4x^2 - 20x + 25 = 0$

Factorizing $(2x - 5)(2x - 5) = 0$

or $(2x - 5)^2 = 0$

For both factors, $2x - 5 = 0$, giving $x = 2.5$

With this equation both roots are the same and we say that the equation has equal roots. This always happens when the expression $ax^2 + bx + c$ forms a perfect square.

Example 4:

Solve $x - 10 + \frac{9}{x} = 0$

Multiply each term by x to clear the fraction,

$x^2 - 10x + 9 = 0$

Factorizing $(x - 9)(x - 1) = 0$

Either $(x - 9) = 0$, giving $x = 9$ or $x - 1 = 0$, giving $x = 1$The roots are $x = 9$ and $x = 1$.

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Equations with no linear term

When in a quadratic equation: $ax^2 + bx + c = 0$ the coefficient b = 0, the equation is in the form: $ax^2 + c = 0$

In this case the solution can be determined by use of the square root.

Example 1:

To solve the quadratic equation: $x^2 - 9 = 0$

you just take to square root of c: $x^2 = 9$

$$x_{1,2} = \pm \sqrt{9}$$

If you take the square root the result can be positive as well as negative.

Example 2:

Solve the equation $2x^2 - 32 = 0$

$$2x^2 = 32$$

$$x^2 = 16$$

$$x = \pm \sqrt{16}$$

and $x = \pm 4$

Example 3:

Solve the equation $2x^2 + 18 = 0$

$$2x^2 = -18$$

$$x^2 = -\frac{18}{2} = -9$$

and $x = \pm \sqrt{-9}$

The square root of a negative quantity has no arithmetic meaning and it is called an imaginary number. The reason is as follows:

$$(-3)^2 = 9$$

$$(+3)^2 = 9$$

$$\sqrt{9} = \pm 3$$

Hence it is not possible to give a meaning to $\sqrt{-9}$. The equation $2x^2 + 18 = 0$ is said, therefore, to having imaginary roots.

Using the Formula to Solve Quadratic Equations

This equation is called the quadratic formula. Note that the whole of the numerator, including $-b$, is divided by $2a$. The formula is used when factorization is not possible, although it may be used to solve any quadratic equation.

Example

Solve the equation $3x^2 - 8x + 4 = 0$

Comparing with $ax^2 + bx + c = 0$ we have $a = 3, b = -8$ and $c = 4$. Substituting these values in the formula gives:

$$x = \frac{\left(-b \pm \sqrt{b^2 - 4ac}\right)}{2a}$$

$$x = \frac{\left(-(-8) \pm \sqrt{(-8)^2 - 4 \times 3 \times 4}\right)}{(2 \times 3)}$$

$$x = \frac{\left(8 \pm \sqrt{64 - 48}\right)}{6}$$

$$x = \frac{\left(8 \pm \sqrt{16}\right)}{6}$$

$$x = \frac{(8 + 4)}{6} \text{ and } x = \frac{(8 - 4)}{6}$$

$$x = 2 \text{ and } \frac{2}{3} = 0.67$$

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Logarithms

Why do we use Logs?

In the discussion of indices it was noted that whenever a number is "raised" to a power then we write that in exponential notation and the meaning of it is that the number appearing in the base is being multiplied by itself the number of times that is indicated by the exponent. The notation used was such that if we write 5^3, what we actually mean is 5 multiplied by itself 3 times.

Logarithms are mathematical inventions in order to answer a slightly different question (notice the word "invention", logarithms make certain operations easier to handle and that is all they do, so you should think of them as a definition).

What number do I have to raise to the power of 3 in order to get 1000? This might seem pretty simple and obvious. If you multiply 10 x 10 you 100, and if you multiply 100 x 10 you get 1000.

So, you would say that 10 multiplied by itself 3 times or, in our power notation, 10^3 is equal to 1000.

Now, this is easy to answer by thinking about powers because the above example is simple powers and simple numbers, and one can reason it out relatively quickly. However, things can get more complicated. Suppose now that you were asked "what number do I have to raise 10 to in order to get 735. All of a sudden the answer is not very obvious. What is so different about this question?

There is actually nothing different about this question. You still can try doing the same process, but now the number is not that pretty and it's not exactly obvious how many times you should multiply 10 by itself to get 735. If you multiply it by itself 2 times you get 100, but 3 times gives 1000 and you have already exceeded 735! How do we "get out" this power that we need.

Logarithms are at the most basic level – invented to answer the general question of how does one extract the base or exponent of an algebraic power when one of these is an unknown.

What are Logs?

Continuing on the above reasoning, let us take our simple example again, what number raised to the power of 3 gives 1000? If we invent an unknown variable called x and write out our question in terms of the notation of algebraic powers we have the following situation:

$$10^x = 1000$$

The question is what is x in the above formula? How do we solve x?
We invent an operation called logarithm, abbreviated to Log and we apply this operation to both sides of the above relation.

$$Log10^x = Log1000$$

We then define this new logarithm function to be such that when it applies to a number that is "10 raised to some power" then it literally just gives us the power and the base 10 "disappears". To reiterate this, what we mean is the following:

$$Log10^x = x$$

So, what is the value of Log 1000? How do we know what the right hand side of the equation is? How does this help us at all? Well, this will probably seem magic to you at this point, but one option is to find the (Log) button on your calculator and take the Log 1000 and you get 3. So, $x = 3$, and it is what we expected.

How does this help us with anything? It seems like we went in a big loop, and we knew the answer to begin with anyway. But, now consider the slightly more complicated question that we had above. What number do I raise 10 to in order to get 735? Let us apply the logarithm process to this situation:

$$Log10^x = Log735$$
$$x = Log735$$

If you take the Log of 735 on your calculator you get 2.866. So, 10 raised to the power of 2.866 gives you 735, and the question is answered. Recall that algebraic powers need not be integers, and we have a clear example of a non–integer power.

In easy words we can say, the logarithm function is the reverse of exponentiation.

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COMMON LOGARITHMS

There are two basic types of logarithms that are important to know. In the previous section, where logarithms were defined, you already saw the definition of one kind of logarithms, that was the so called Log Base 10.The logarithmic operation that we have introduced serves the main purpose of extracting the exponents in an algebraic power. This is true of the operation of “taking the logarithm”.

Logarithm Base 10

The logarithm of base 10 is most often useful when powers of 10 are involved, but not necessarily. It can be used in many other situations. For instance, suppose you were asked the following question: 3 raised to what power gives 16.8? Again, applying our definition of logarithm of base 10 as defined in the previous section we can answer the question but, in order to do this we need to define some rules of operation for logarithms.

You can think if Logarithm Base 10 as the logarithmic operation that when carried out on 10 raised to some power ends up giving us the power. The log of base 10 is written as : Log_{10} . Thus, $Log_{10}(10^x) = x$. This is the basic definition of base 10 logs. abbreviated to “lg”.

The following values may be checked using a calculator:

lg 15.4 = 1.1875

lg 378.2 = 2.577

lg 0.0241 = −1.6179.

Natural Logarithms

There is another logarithm that is also useful (and in fact more common in natural processes). Many natural phenomenons are seen to exhibit changes that are either exponentially decaying (radioactive decay for instance) or exponentially increasing (population growth for example).

These exponentially changing functions are written as e^x, where x represents the rate of the exponential.

In such cases where exponential changes are involved we usually use another kind of logarithm called natural logarithm. The natural log can be thought of a logarithm base−e (where ’e’ is a mathematical constant approximately equal to 2.7183, also called Euler’s number). What this means is that it is a logarithmic operation that when carried out on e raised to some power gives us the power itself.

Those logarithms are called hyperbolic or natural logarithms Log_e and are labelled with Ln (for “logarithm naturalis”) and its definition is:

$\mathrm{Ln}(e^x) = x$.

The following values may be checked by using a calculator:

ln 4.73 = 1.5539

ln 278.4 = 5.6290

ln 0.7642 = −0.2689

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RULES OF LOGARITHM

There are three rules of logarithms, which apply to any base.

Rule 1.

To multiply two numbers

$$LogAB = LogA + LogB$$

Example:

$$Log_{10}10^1 = 1$$

Also $Log_{10}(10^3 \times 10^5) = Log_{10}10^8 = 8$

Hence $Log_{10}10^3 + Log_{10}10^5 = 3 + 5 = 8$

Rule 2.

To divide two numbers

$$Log\frac{A}{B} = LogA - LogB$$

Example:

$$Log_{10}10^8 - Log_{10}10^5 = Log_{10}10^3$$

Also $8 - 5 = 3$

Hence $Log_{10}\frac{10^8}{10^5} = Log10^3_{10} = 3$

Rule 3.

To raise a number to a power

$$LogA^n = nLogA$$

Example:

$$Log_{10}10^5 = 5$$

Also $5 \times 1 = 5$

Hence $5 \times Log_{10}10^1 = 5 \times 1 = 5$

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Numbering Systems

Various Numbering Systems

In mathematics, various numbering systems can be used.

In most parts of Europe the Roman numeral system was used. Instead of digits the letters I, V, X, L, C, D and M were used. In the 14th century the Arabic numerals with the digits 1 to 9 were used, along with the 0.

The system which is now used in most parts of the world is the decimal numeral system which is based on the Arabic numerals.

The development of computer technology required other numbering systems.

For this reason, the following numbering systems are in use:

- decimal numeral system
- binary numeral system
- octal numeral system
- hexadecimal numeral system.

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Decimal Numeral System

The decimal numeral system is the system which is commonly used.

The name descends from the Latin decem = ten.

But for the programming of computers, this is unusable.

In the decimal numeral system, the significance of each digit's position is related to a power of ten.

Powers of 10

Powers of 10:	10^5	10^4	10^3	10^2	10^1	10^0
Decimal numbers:	100000	10000	1000	100	10	1

Negative Indices

It is also possible to use negative indices.

10^{-1} equates to $\frac{1}{10}$.

10^{-2} equates to $\frac{1}{10^2}$.

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Binary Numeral System

General

Every number that can be written in decimal can also be written in another system called Binary. Binary is the main number systems used by computer scientists.

The binary number system is a base 2 number system which uses only the digits 0 and 1. It is also a place value system which means that each place represents a power of 2, just as the place represents a power of 10 in the decimal system.

Powers of 2

Powers of 2:	2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}
Decimal No:	32	16	8	4	2	1	.	0.5	0.25
eg: 1010.01_2	0	0	1	0	1	0	.	0	1

The number 1010.01_2 therefore means:

1 x 8 = 8
+1 x 2 = 2
+1 X 0.25 = 0.25

So

$1010.01_2 = 10.25_{10}$

Notation

If the context does not give the clear information about which numeral system is being used the numeral system is exposed by a subscript.

101_{10} means that the “normal“ decimal numeral system is used.
It means one hundred and one.

101_2 means that the binary numeral system is used.
It means once 2^2, no 2^1 and once 2^0.

Sometimes the digits are written in groups by four to get a clear arrangement.

Binary to Decimal Conversions

Example 1:

Convert 1010 to a Decimal

Write down the powers of 2 and add them to a sum.

$1 \times 2^3 = 8$
$+ 0 \times 2^2 = 0$
$+ 1 \times 2^1 = 2$
$+ 0 \times 2^0 = 0$
10

Thus

1010_2 is equal to 10_{10}

Example 2:

Convert 1001001 to a Decimal

Write down the powers of 2, and the number to be converted below them, as follows:

64	32	16	8	4	2	1
1	0	0	1	0	0	1

Then add all the numbers above the 1’s

i.e. 64 + 8 + 1 = 73

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Decimal to Binary Conversions

Every figure in the decimal numeral system can be broken down into powers of 2 and thus can be transferred to the binary numeral system.

Method:

Convert 271 to Binary

Find the powers of two which is equal or smaller than the number to be converted.

For **271** this is 2^8, that equals 256_{10}.

256 128 64 32 16 8 4 2 1

2^8 = 256 is one time in the **271**.

Write a **1** under the 256.

Subtract 256 from **271**.

271 – 256 = 15

2^7 is no time in the 15, so note a **0**..

The same is with 2^6, 2^5 and 2^4. Here you also note a **0** for each.

Then 2^3 = 8 is one time in the 15, so you note a **1**, please.

Subtract 8 from 15.

15 - 8 = 7

2^2 = 4 is one time in the 7, so you note a **1**.

Subtract 4 from 7.

7 - 4 = 3

2^1 = 2 is one time in the 3, so you note a **1**.

Subtract 2 from 3.

3 - 2 = 1

2^0 = 1 is one time in the 1, so you note a **1**.

The result of all the **1** and **0** is: **100001111**

If you write all the powers of 2 in a line and then the zeros and ones below, you get the following result:

First write all powers of two, beginning with the highest number which has to be used for the calculation. Again, 256 in this case.

256	128	64	32	16	8	4	2	1
1	0	0	0	0	1	1	1	1

The decimal number 271_{10} is therefore 100001111_2

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Octal Numeral System

General

Every number that can be written in decimal can also be written in another system called octal. Like binary, octal is one of the three main number systems used by computer scientists.

The octal number system is base 8 number system which uses only the eight digits 0, 1, 2, 3, 4, 5, 6, and 7. It is also a place value system which means that each place represents a power of 8, just as the place represents a power of 10 in the decimal system.

Powers of 8

Powers of 8;	8^4	8^3	8^2	8^1	8^0
Decimal No:	4096	512	64	8	1

It is possible to show values which are smaller than 1:

8^{-1} equals 0.125 in the decimal system.

8^{-2} equals 0.015625 in the decimal system.

Notation

This numeral system can be exposed by a subscript, too.

101_8 means that this is the octal numeral system. It means one time 8^2, no 8^1 and one time 8^0.

Sometimes the digits are written in groups by four to get a clear arrangement.

Octal to Decimal Conversion

Convert 237 to a Decimal

Powers of 8:	8^4	8^3	8^2	8^1	8^0
Decimal numbers:	4096	512	64	8	1
eg: 237_8			2	3	7

Thus, an octal number such as:

$$237_8 = 2 \times 8^2 + 3 \times 8^1 + 7 \times 8^0 = 128 + 24 + 7 = 159_{10}$$

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Converting Binary to Octal

To convert a binary number to an octal number, construct a 3–bit binary / octal lookup table like the one below. Starting at the binary decimal point of the binary number, take the first 3 bits and find the corresponding octal value from the table.

3–bit binary / octal table

3–bit Binary	Octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Start with the 3 most right bits and find the related octal value in the table.

Repeat this with the next 3 bits and so on. If there are less than 3 bits left, fill with zeroes until you have 3 bits. Again, use the table.

Example:

Convert 11010010 to octal.

1. Take the 3 most right bits, 010 and find the corresponding octal value in the above lookup table. The octal value is 2.
2. Take the next 3 bits, 010. The corresponding octal value from the lookup table is 2 again.
3. Now only 2 bits, 11 of the binary number remain. Pad the left hand side with a 0 to get 011. The corresponding octal from the lookup table is 3.

so, $11010010_2 = 322_8$

Converting Octal to Binary

For the conversion of octal numbers to binary numbers, just use the table in a reverse manner.

To convert from octal to binary, write down the binary representation of each octal digit. Note that each octal digit should take up 3 bits.

3–bit Binary	Octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

3–bit Binary/Octal table

Example:

Convert 322_8 to binary

3 = 011

2 = 010

2 = 010

so, $322_8 = 011010010_2$

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Hexadecimal Numeral System

General

Every number that can be written in decimal can also be written in another system called hexadecimal. Hexadecimal is the last of the three main number systems used by computer scientists.

The hexadecimal number system is a base 16 number system which uses the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, & F. Here, we need the extra digits A, B, C, D, E and F to represent the numbers 10, 11, 12, 13, 14, and 15, since there are no digits in the decimal numeral system to do this.

Decimal/Hexadecimal Table

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

Hexadecimal is also a place value system which means that each place represents a power of 16, just as the place represents a power of 10 in the decimal system:

Powers of 16:	16^3	16^2	16^1	16^0	.	16^{-1}
Decimal No:	4096	256	16	1	.	0.0625
eg.: $3AF_{16}$ = 3	A	F				

It is possible to show values which are smaller than 1:

16^{-1} equals 0.0625 in the decimal system.

Technology

Some aircraft systems display information in an hexadecimal code. This data must be converted to the binary system then.

Example:

Convert 3AF to a decimal number

Powers of 16:	16^3	16^2	16^1	16^0	.	16^{-1}
Decimal No:	4096	256	16	1	.	0.0625
eg.: $3AF_{16}$ =		3	A	F		

Thus, a hexadecimal number such as:

$$3AF_{16} = 3 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 = 768 + 160 + 15 = 943_{10}$$

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Converting Binary to Hexadecimal

To convert a binary number to an hexadecimal number, construct a 4−bit binary/hexadecimal lookup table like the one below. Starting at the binary decimal point of the binary number, take the first 4 bits and find the corresponding hexadecimal value from the table. Repeat with the next 4 bits and so on. If there is less than 4 bits remaining, pad them out to 4 bits.

4−bit Binary/Hexadecimal Table

4−bit Binary	Hexadecimal
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Example:

Convert 11010010 to Hexadecimal

1. Take the 4 most right bits, 0010 and find the corresponding hexadecimal value in the lookup table. The hexadecimal value is 2.
2. The next 4 bits, 1101 and find the corresponding hexadecimal value in the lookup table. The hexadecimal value is D.

so, $11010010_2 = D2_{16}$

Converting Hexadecimal to Binary

To convert from hexadecimal to binary, write down the binary representation of each hexadecimal digit.

Note that each hexadecimal digit should take up 4 bits.

Example:

Convert $2CF_{16}$ to binary

2 = 0010

C = 1100

F = 1111

so, $2CF_{16} = 001011001111_2$

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M1.3 GEOMETRY

Geometrical Constructions

TRIANGLES

Pythagoras Theorem

"In a right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides".

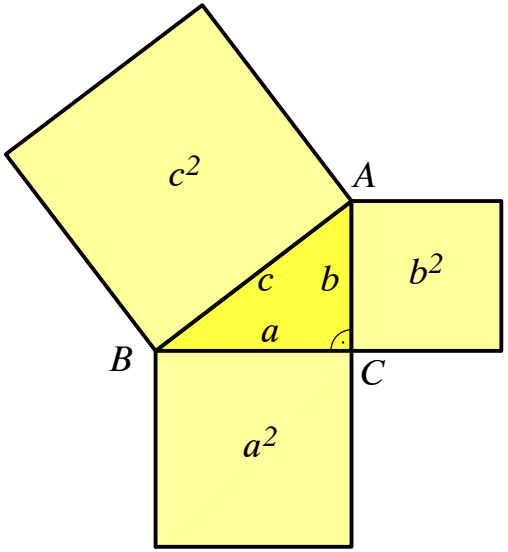


Figure 14 Pythagoras Theorem

For the triangle shown:

$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$

$b^2 = c^2 - a^2$

Pythagoras Theorem is particularly useful to you when dealing with vectors in aircraft theory.

Example:

A right angled triangle has smaller sides of length 5 cm and 12 cm. What is the length of the hypotenuse?

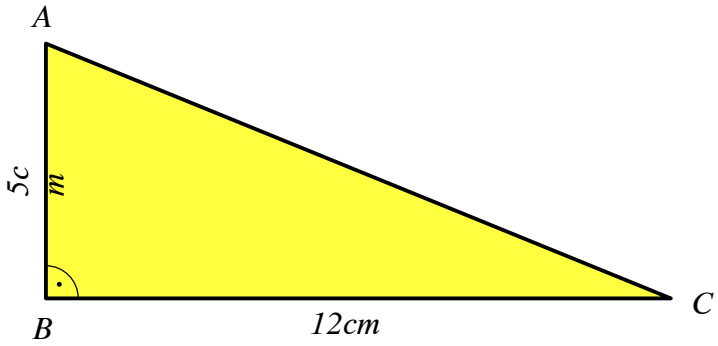


Figure 15 Pythagoras Example 1

$= 144 + 25$

$= 169$

$c = \sqrt{169}$

$= \mathbf{13\ cm}$

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Pythagoras Theorem Application

Pythagoras Theorem can be used to find the height of equilateral and isosceles triangles (an equilateral triangle has all its sides of equal length and an isosceles triangle has 2 sides of equal length).

Example

Find the height of an isosceles triangle which has sides of length 13 cm and base of length 10 cm.

$h^2 = 13^2 - 5^2$

$= 169 - 25$

$h^2 = 144$

$h = 12cm$

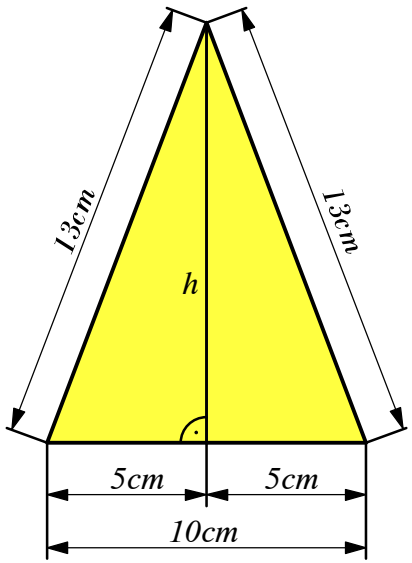


Figure 16 Pythagoras Example 2

NOTE: The height is always drawn at right angles to the base and goes to the opposite apex. We can draw the height from any side providing it meets the above requirement, i.e it cuts the chosen base at right angles and goes from the base to the opposite apex.

As an example, we could tip the triangle onto its side and take a new height for it.

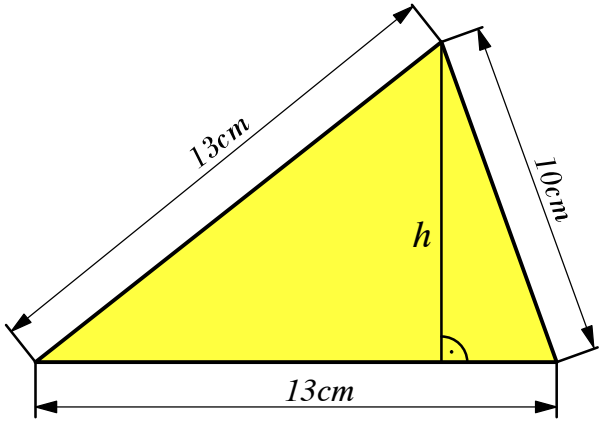


Figure 17 Pythagoras Example 3

NOTE: Now that the base is 13 cm long, whereas before 10 cm long. The height will also be a different length, so there is not one height for one triangle, it all depends on which side you use as your base.

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Trigonometry

Trigonometrical Ratios

By using Pythagoras, you are now able to partially solve right−angled triangles, e.g. you can find the third side of a right−angled triangle when given its other 2 sides. This chapter is concerned with establishing the basic trigonometrical concepts which will later enable you to completely solve right−angled triangles, i.e. to find all their 6 elements (angles and sides).

Similar triangles, are triangles which have the same shape, one is simply an enlargement of the other. Two important properties of similar triangles are:

1. their corresponding angles are equal
2. their corresponding sides are proportional

Consider the triangles:

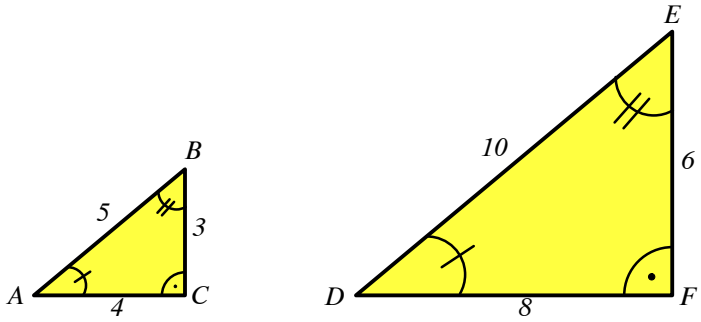


Figure 18 Equiangular Triangles

The above triangles are similar since they are equiangular and the ratios of their corresponding sides are constant, i.e.

1. $\frac{BC}{AB} = \frac{3}{5} = \frac{EF}{DE} = \frac{6}{10} = \frac{3}{5}$
2. $\frac{AC}{AB} = \frac{4}{5} = \frac{DF}{DE} = \frac{8}{10} = \frac{4}{5}$
3. $\frac{BC}{AC} = \frac{3}{4} = \frac{EF}{DF} = \frac{6}{8} = \frac{3}{4}$

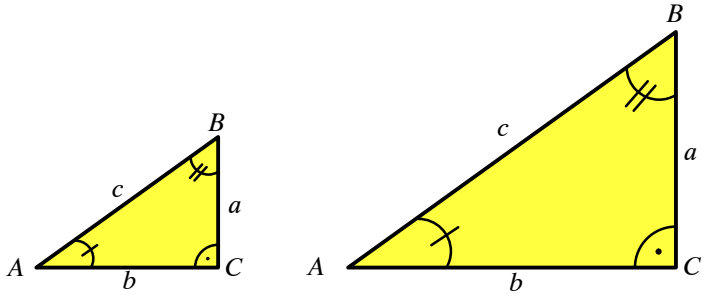


Figure 19 Triangle Side Ratio

Taking angle A as the reference:

1. Side 'a' is the side opposite
2. Side 'b' is the adjacent

Taking angle B as the reference:

1. Side 'b' is the side opposite
2. Side 'a' is the adjacent side

Since the triangles are similar, the ratios of corresponding sides are constant, i.e. the ratios $\frac{a}{c}$, $\frac{b}{c}$ and $\frac{a}{b}$ are the same for all similar right−angled triangles.

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Ratio

In a right−angled triangle the ratio:

Sine

1. $\frac{Side\ opposite\ the\ Angle}{Hypotenuse}$ is called the Sine of the reference angle

$\therefore \sin A = \frac{opposite}{hypotenuse} = \frac{a}{c}$

Cosine

2. $\frac{Side\ adjacent\ the\ Angle}{Hypotenuse}$ is called the Cosine of the reference angle

$\therefore \cos A = \frac{adjacent}{hypotenuse} = \frac{b}{c}$

Tangent

3. $\frac{Side\ opposite\ the\ Angle}{Side\ adjacent\ the\ Angle}$ is called the Tangent of the angle

$\therefore \tan A = \frac{opposite}{adjacent} = \frac{a}{b}$

The above are the fundamental trigonometrical ratios for right−angled triangles and must be remembered. A convenient method to help you to remember them is SOHCAHTOA, where S = sin, C = cos and T = tan.

Example:

For the triangle shown:

1. sine of angle B
2. cosine of angle B
3. tangent of angle B

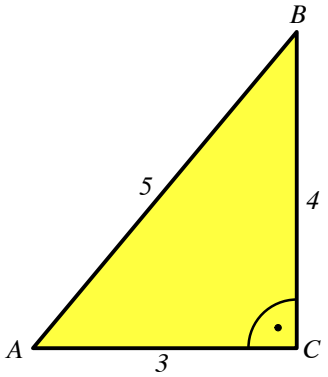


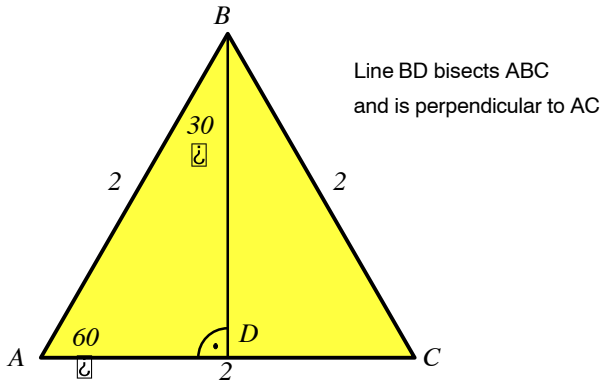
Figure 20 Angle Function Example

1. $\sin B = \frac{opp}{hyp} = \frac{3}{5} = 0.6$
2. $\cos B = \frac{adj}{hyp} = \frac{4}{5} = 0.8$
3. $\tan B = \frac{opp}{adj} = \frac{3}{4} = 0.75$

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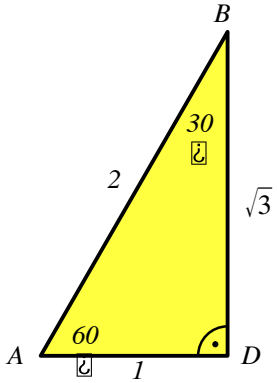
We will now calculate the values for 30° and 60°. Consider the equilateral triangle ABC of sides 2 units.



Line BD bisects ABC
and is perpendicular to AC

Figure 21 Equilateral Triangle

In triangle ABD, ∠A = 60°, ∠B = 30° and ∠D = 90°



side d = 2 (given)
side b = 1 (half of AC)
side $a^2 = 2^2 - 1^2$
$a^2 = 3$

Figure 22 Calculation

Thus in a right angled triangle ABD:

1. $\sin 60° = \frac{opp}{hyp} = \frac{\sqrt{3}}{2} = \frac{1.7321}{2} = 0.8660$
2. $\cos 60° = \frac{adj}{hyp} = \frac{1}{2} = 0.5000$
3. $\tan 60° = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = 1.732$
4. $\sin 30° = \frac{opp}{hyp} = \frac{1}{2} = 0.5000$
5. $\cos 30° = \frac{adj}{hyp} = \frac{\sqrt{3}}{2} = \frac{1.7321}{2} = 0.8660$
6. $\tan 30° = \frac{opp}{adj} = \frac{1}{\sqrt{3}} = 0.5774$

We can now collect all our information and show graphically how the basic trigonometrical ratios change as the angle increases from zero to 90°.

The functions all give graphs which are important. You should know how to sketch them and know how to use them.

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Example:

The following example involves the use of trigonometry, or combinations of trigonometry and Pythagoras' theorem, to solve right−angled triangles.

In the right−angled triangle ABC, find angle A and side c.

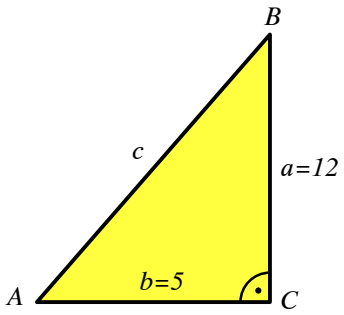


Figure 23 Trigonometry and Pythagoras Combination

Angle A

Since $\frac{opp}{adj}$ = tan, this is the ratio we use

$\therefore$ tan A = $\frac{opp}{adj}$

tan A = $\frac{12}{5}$

tan A = 2.4

A = 67°.23 (after using a calculator)

Side c

by Pythagoras' theorem,

$\therefore$ $c^2 = a^2 + b^2$

$c = \sqrt{12^2 + 5^2}$

$c = \sqrt{169} = 13$

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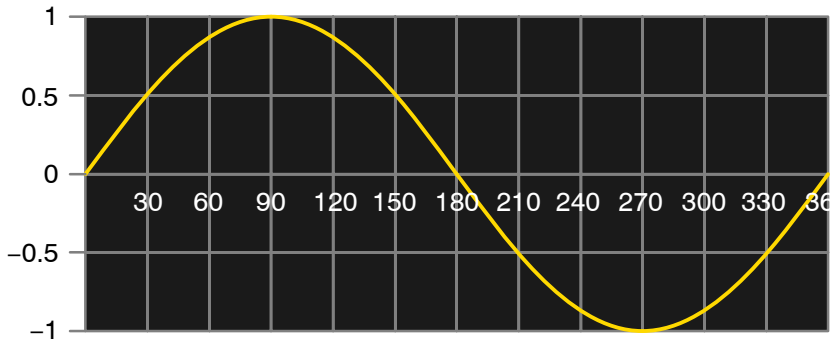


Figure 24 Sine Curve

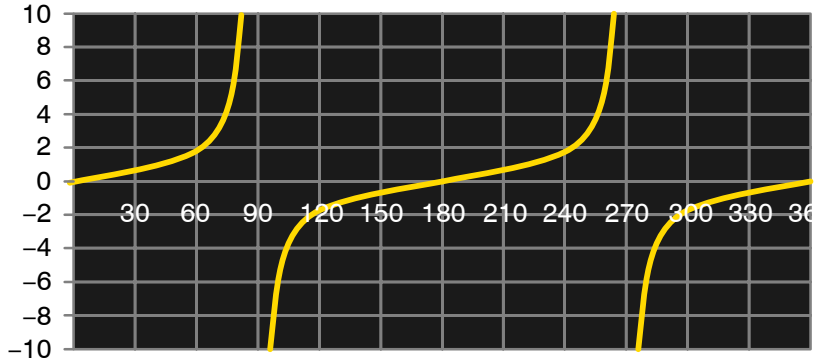


Figure 26 Tangent Curve



Figure 25 Cosine Curve

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Trigonometric Tables

General

Sine and cosine can be used to calculate the side of a rectangular triangle if the hypotenuse is known and the angle is known.

To find the sine or cosine which corresponds to the angle, tables were developed and published.

They were used before calculators became common fashion.

A table is shown here.

α [°]	sin α	cos α
0	0,000	1,000
5	0,087	0,996
10	0,174	0,985
15	0,259	0,966
20	0,342	0,940
25	0,423	0,906
30	0,500	0,866
35	0,574	0,819
40	0,643	0,766
45	0,707	0,707
50	0,766	0,643
55	0,819	0,574
60	0,866	0,500
65	0,906	0,423
70	0,940	0,342
75	0,966	0,259
80	0,985	0,174
85	0,996	0,087
90	1,000	0,000

Example of a Triangle

The angle α is the angle that is formed by the sides b and c.

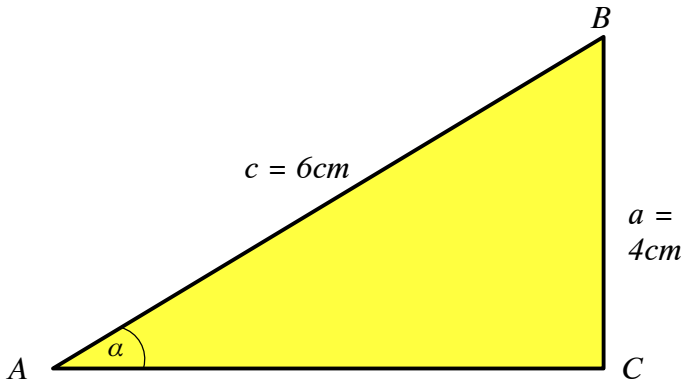


Figure 27 Angle Function Example

Side „a“ has a length of 4 cm and is located opposite the angle α. Side c has a length of 6 cm and is located opposite the right angle. Therefore it is called the hypotenuse. In accordance with the trigonometric ratios, sine is defined by the side opposite the angle over the hypotenuse.

$$\sin\alpha = \frac{opp}{hyp} = \frac{4cm}{6cm}$$

$$\sin\alpha = 0.667$$

Now take the result of 0.667 and look at the sin α column of the table. You will see that the angle α is slightly bigger than 40°, which is correct.

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Example 1:

In technical calculations, trigonometry is used very often.

An example is to calculate the friction on a plane ground.

A steel body weighing 450 Newton (450 N) is resting on a horizontal steel surface. How many newtons of force are necessary to start the body sliding?

Weight (W) = 450 N

The friction coefficient for these materials μ = 0.15

Force F = $\mu \times$ W

Force to start sliding motion F = 0.15 $\times$ 450 N = 68 N

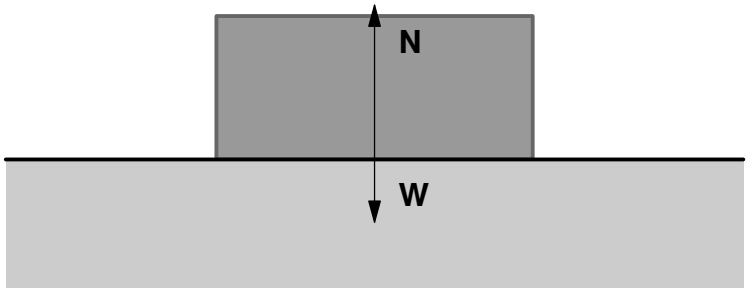


Figure 28 Plane Ground Friction Example

But this is only true if the plane has no decline.

Example 2:

The correct formula uses trigonometry. By this, frictions can also be calculated if the plane is inclined.

The complete formula is:

F = $\mu \times$ W $\times$ cos α

In our example the angle is 20°.

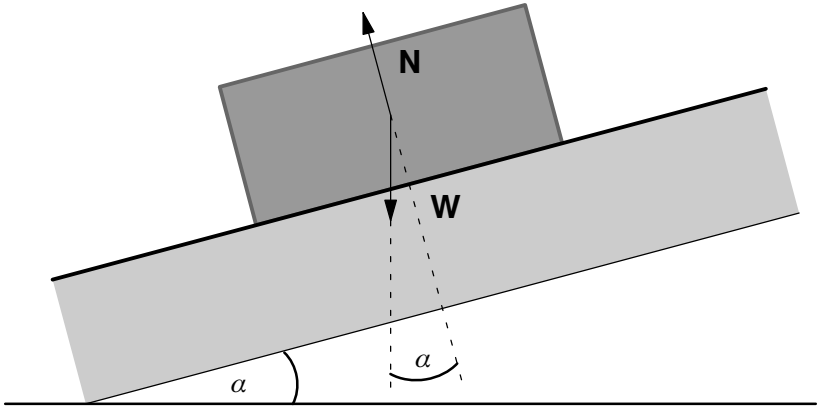


Figure 29 Plane Ground Friction Example

According to the table, the cosine of 20° is 0.94.

Force to start sliding motion F = 0.15 $\times$ 0.94 $\times$ 450 N = 63 N

Summary

The bigger the angle α, the smaller cos α.

At an angle of 45° the force needed to move the body is only 48 N.

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Coordinates and Graphs

Coordinates

An equation involving two variables can be represented by a graph drawn on "coordinate axes".

Coordinate axes (see below) consist of a horizontal line (referred to as the x axis) and a vertical line (referred to as the y axis). The point of intersection of these two lines is called the origin (denoted by the letter "O").

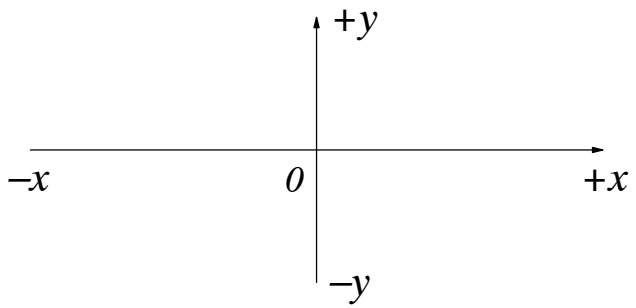


Figure 30 Coordinate Axes

Along the x and y axes we can mark off units of measurement (not necessarily the same on both axes). The origin takes the value zero on both axes. The x axis takes positive values to the right of the origin and negative values to the left of the origin. The y axis takes positive values above the origin and negative values below the origin.

Any point on this diagram can be defined by its coordinates (consisting of two numbers). The first, the x coordinate, is defined as the horizontal distance of the point from the y axis, the second, the y coordinate, is defined as the vertical distance of the point from the x axis.

In general, a point is defined by its coordinates which are written in the form (a, b).

Example:

The point (3, 2) may be plotted on the coordinate axes as follows:

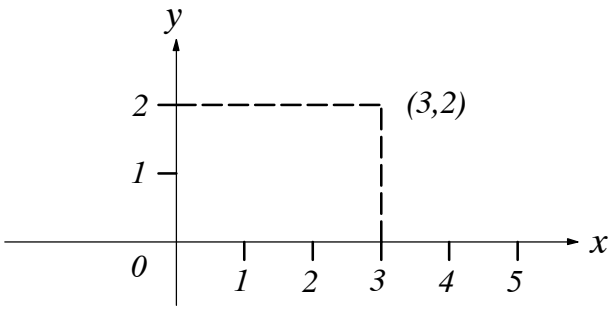


Figure 31 Coordinate Point

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Graphs

An equation involving two variables can be represented, on coordinate axes, by means of a graph.

For a given range of values of x, the corresponding y values can be calculated from the equation being considered. The points obtained can then be plotted and joined together to form the graph.

Axes

Before plotting the points on a graph, the axes must be drawn in a way that takes into account the range of the x–values and the range of the y–values.

Scale

If graph is used (which is desirable) you should use a scale that involves a sensible number of units per square i.e. you should use steps of, for example, 1, 2, 5 or 10 etc. units per square depending on the question.

You should avoid using steps along the axes of, for example 7 or 9 units per square as this can complicate the graph unnecessarily.

Example:

Draw the graph of y = 2x + 1 between x = 0 and x = 5

By taking the x values 0, 1, 2,...5, we can calculate the corresponding y values, as shown below, by first evaluating the component parts of the equation.

x:	0	1	2	3	4	5
2x:	0	2	4	6	8	10
+1	1	1	1	1	1	1
y:	1	3	5	7	9	11

We then plot the points obtained, each point being defined by its x coordinate and its corresponding y coordinate. The points are then joined together to the graph.

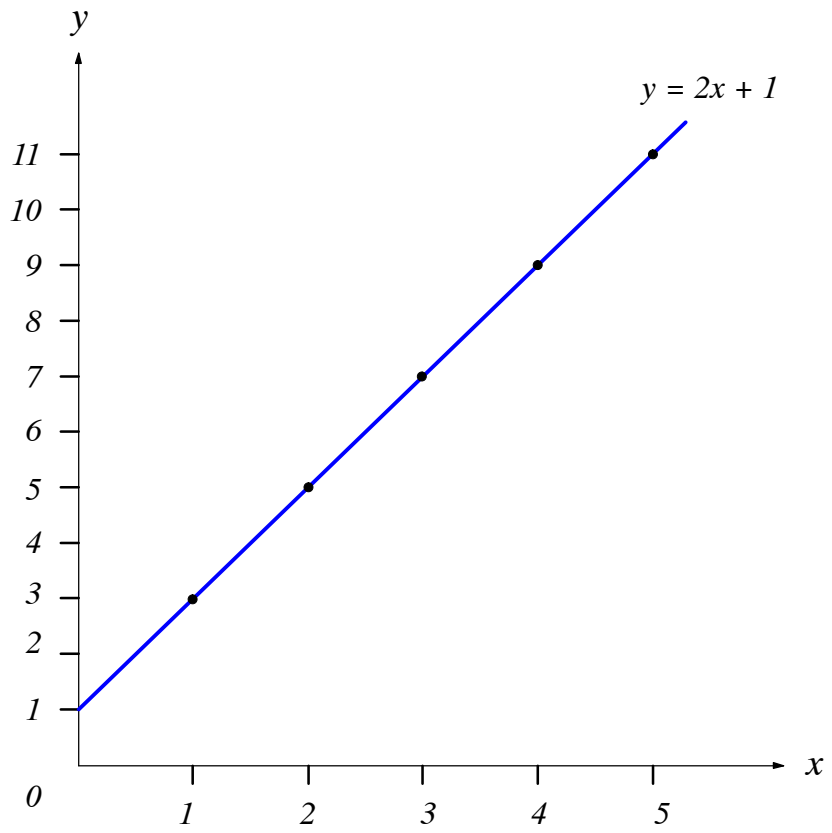


Figure 32 Graph Example

Equations of the First Degree

Equations of the type y = 2x + 1, where the highest powers of the variables, x and y, are the first are called equations of the first degree. All equations of this type give graphs which are straight lines and hence they are often called linear equations.

In order to draw graphs of linear equations we need only two points, however three points are advisable.

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The Straight Line Graph

A straight line is an infinite line. In a plane, its way is defined by two points.

The equation of a straight line is given by:

$$y = mx + c$$

Where **m** represents the slope of the line and **c** is the point where the line crosses the y axis (the y intercept). The point where the line crosses the x axis is called the x intercept.

m = gradient of the line

c = intercept on the y axis

Example 1:

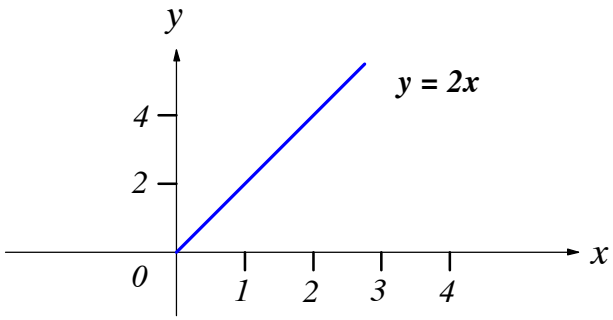


Figure 33 Graph Example (y = 2x)

In this example m = 2 and c = 0

Note: Whenever c = 0 the line will pass through the origin.

Example 2:

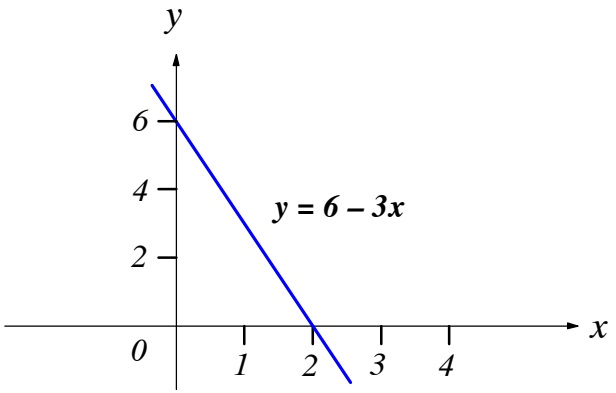


Figure 34 Graph Example (y = 6 – 3x)

In this example m = –3 and c = 6.

As c = 6, we know that this line cuts the y axis at y = 6 (this can be verified by substituting x = 0 into the equation of the line, as x = 0 along the y axis)

Similarly, as y = 0 along the x axis, we can substitute y = 0 into the equation of the line to find where the intersects with the axis (the intercept).

we have, when

$$y = 0$$
$$6 - 3x = 0$$
$$3x = 6$$
$$x = 2$$

Hence the line cuts the x axis at x = 2. We can now say that the y intercept = 6 and the x intercept = 2.

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Example 3:

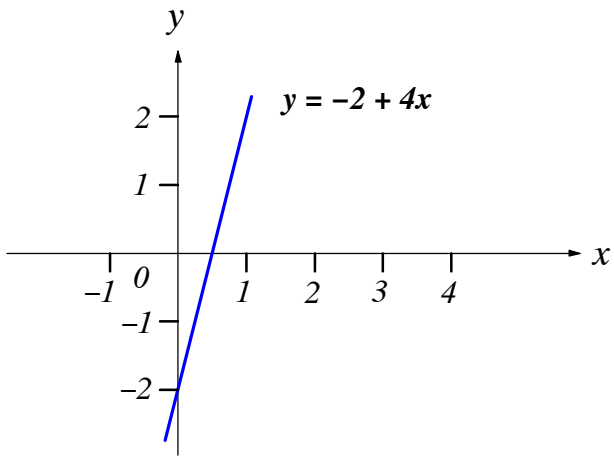


Figure 35 Graph Example (y = −2 + 4x)

In this example, m = 4 and c = −2.

We know immediately that the y-intercept is −2 (the value of c). To find the x intercept, we substitute y = 0 into the equation of the line.

$$0 = -2 + 4x$$
$$4x = 2$$
$$x = 0.5$$

Hence the x intercept is x = 0.5.

A straight line parallel to the x axis takes the form y = constant.

Similarly, a straight line parallel to the y axis takes the form x = constant.

These cases are illustrated below:

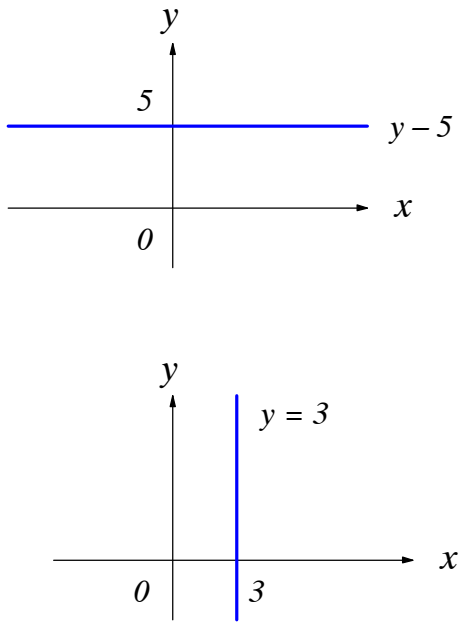


Figure 36 Constant Straight Lines

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Derivation of the Equation of a Straight Line Graph

Given the coordinates of two points (x_1, y_1) and (x_2, y_2) say, we can calculate the equation of the straight line that passes through these points.

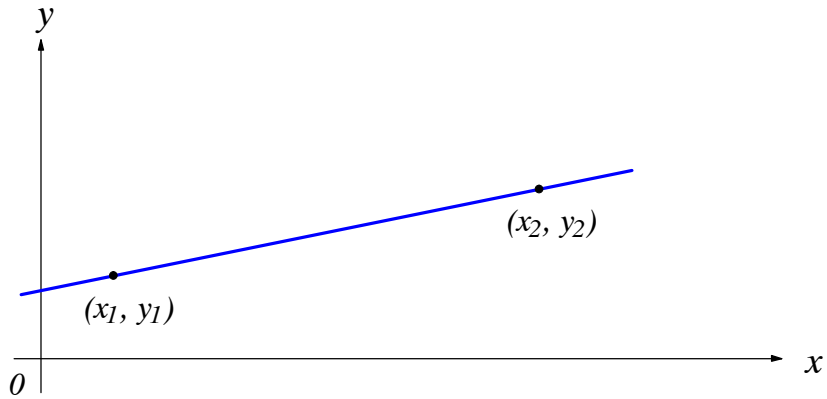


Figure 37 Straight Line Graph Equation Derivation

There are two methods of calculating this equation.

Example:

The question is:

Find the equation of the straight line that passes through the points (1, 4) and 3, 10).

Method 1

The general equation of a straight line is given by y = mx + c and it is necessary to find numerical values for m and c.

If the straight line in question passes through the two given points, then each of these points must satisfy the equation of this straight line. That is, we can substitute the coordinates of each point as follows:

y = mx + c

4 = m + c (1)

Likewise, substituting (3, 10) we have

10 = 3m + c (2)

Now (1) and (2) give us equations in two unknowns, m and c, (simultaneous equations) which we can solve.

We have

4 = m + c (1)

10 = 3m + c (2)

Subtracting (1) from (2) to eliminate c we obtain 6 = 2m

m = 3

Substituting this value of m back into (1) we obtain

4 = m + c

4 = 3 + c

c = 4 – 3

c = 1

If we now substitute these numerical values of m and c into the equation y = mx + c, we obtain the equation of the straight line passing through the points (1, 4) and (3, 10).

That is:

y = 3x + 1

Method 2

In general, we can consider any two points (x_1, y_1) and (x_2, y_2). The straight line passing through these points can be written as:

$$y - y_1 = m \times (x - x_1)$$

where: $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$ (m is the gradient of the line)

Applying this to points (1, 4) and (3, 10) we have:

$x_1 = 1, y_1 = 4$

$x_2 = 3, y_2 = 10$

and we hence obtain:

$$m = \frac{10 - 4}{3 - 1} = \frac{6}{2} = 3$$

and our line becomes:

$$y - 4 = 3 \cdot (x - 1)$$

$$y - 4 = 3x - 3$$

$$y = 3x + 1$$

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Use of Graphs

Nature of Graphs

Graphs normally show two values. Starting in the lower left corner, the x axis shows one value and the y axis shows an other value.

The points of interest are shown in a line or a curve. So for each value on the x axis which is on the curve there is one value of the y axis.

Use of Graphs

When there is a function, for example from a formula, the related curve is shown in a graph.

If somebody has the value for the x axis and needs the value for the y axis, he just reads the graph and reads the y value. No calculation is needed.

Example:

For the servicing of an accumulator with dry nitrogen, limits must be obeyed.

Too low pressure will make the accumulator unserviceable, too high pressure will damage it.

The limits are shown in a graph.

The graph on the right side is taken from the Aircraft Maintenance Manual of a Boeing 757.

The x-axis shows the environmental temperature in degrees Celsius. For each temperature, there are 2 limits. A lower limit and a higher limit.

The pressure is given in PSI.

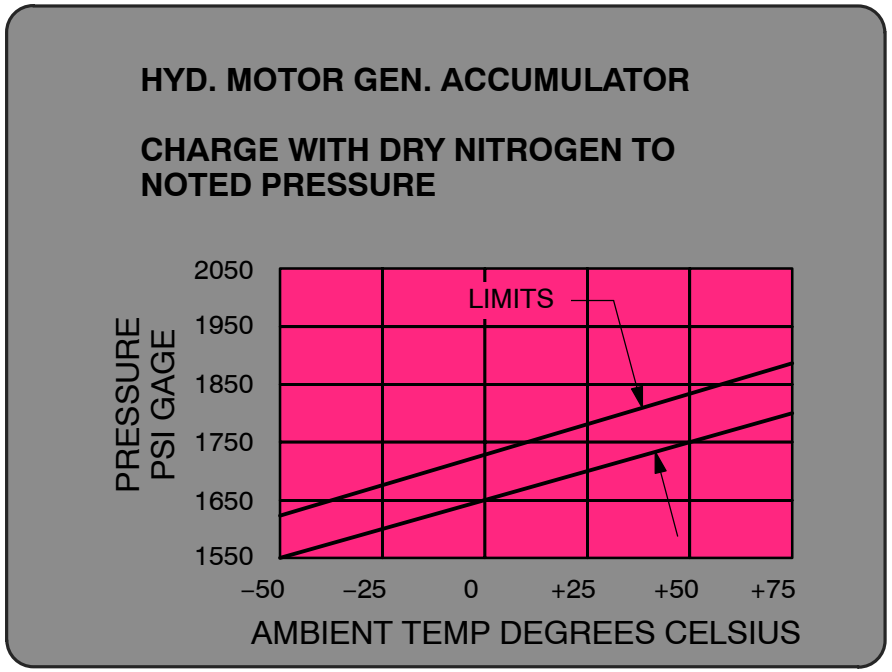


Figure 38 Servicing Placard

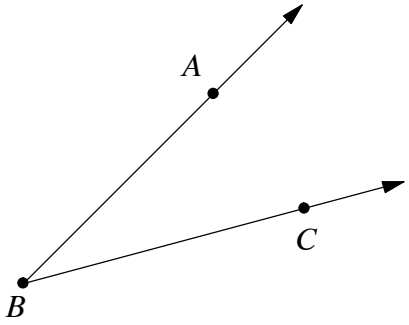
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Angles

General

We can specify an angle by using a point on each ray and the vertex. The angle below may be specified as angle ABC or as angle CBA; you may also see this written as ∠ABC or as ∠CBA how the vertex point is always given in the middle.



Many different names exist for the same angle. For the angle below, ∠PBC, ∠PBW, ∠CBP and ∠WBA are all names for the same angle.

Degrees & Radians–Measuring Angles

We measure the size of angle using degrees. We can also use radians to measure angles.

There are 2π radians in 360°.

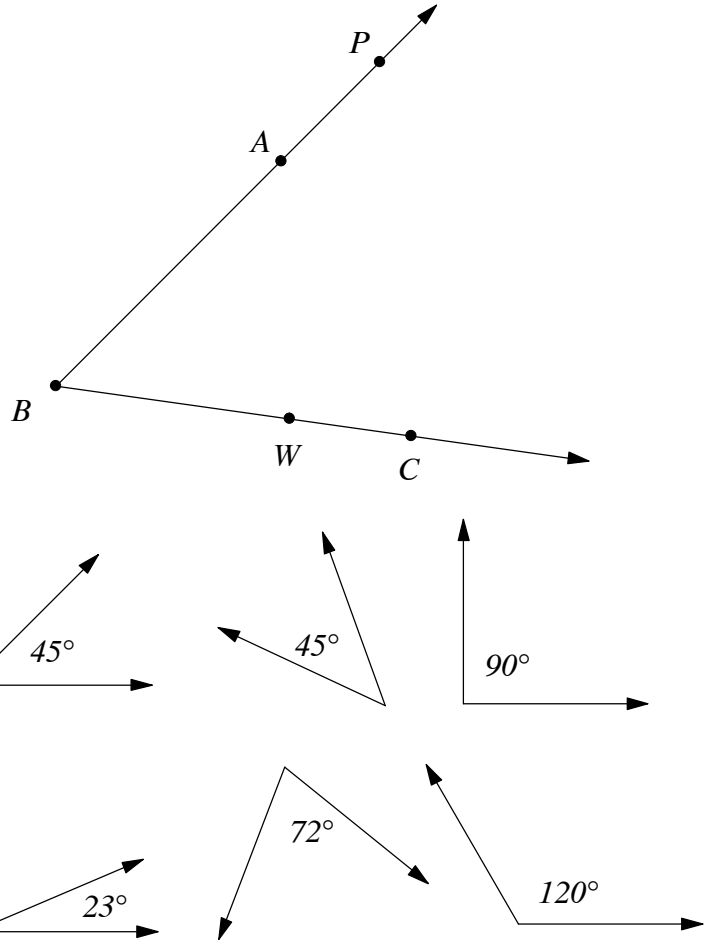
The radius of a circle fits around the circumference 6.26 (or 2π) times.

1 radian = 57.3 degrees. To convert from degrees to radians, use:

$\frac{n^\circ}{360} \times 2\pi$, where n° is the number of degrees.

Example:

Here are some examples of angles and their degree measurements. Convert them to radians.



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Acute Angles

An acute angle is an angle measuring between 0° and 90°.

Example:

The following angles are all acute angles:

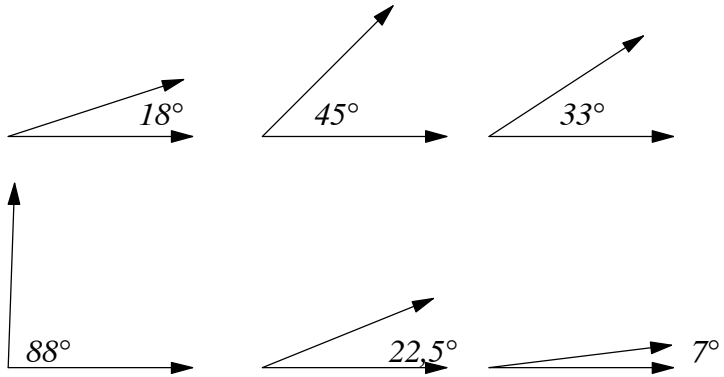


Figure 39 Acute Angles

Obtuse Angles

An obtuse angle is an angle measuring between 90° and 180°.

Example:

The following angles are all obtuse

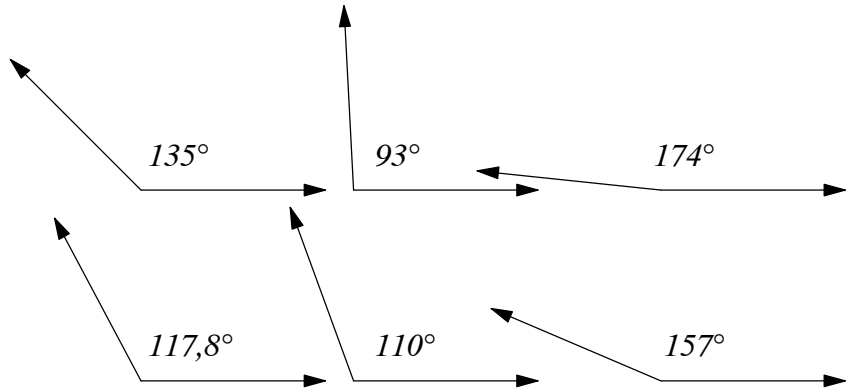


Figure 40 Obtuse Angles

Reflex Angles

A reflex angle is an angle measuring between 180° and 360° degrees.

Right Angles

A right angle is an angle measuring 90°. Two lines or line segments that meet at right angle are said to be perpendicular. Note that any two right angles are supplementary angles (a right angle is its own angle supplement).

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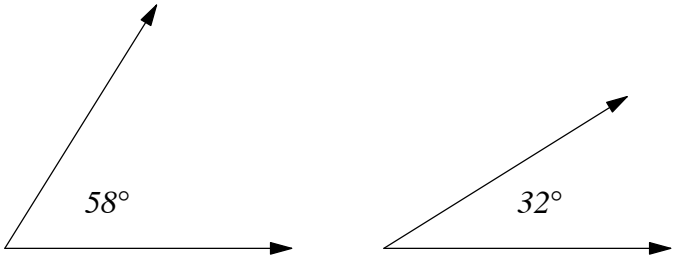
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Complementary Angles

Two angles are called complementary angles if the sum of their degree measurements equal 90°. One of the complementary angles is said to be the complement of the other.

Example:

These two angles are complementary



Note that these two angles can be “pasted” together to form a right angle.

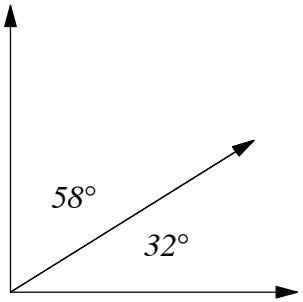


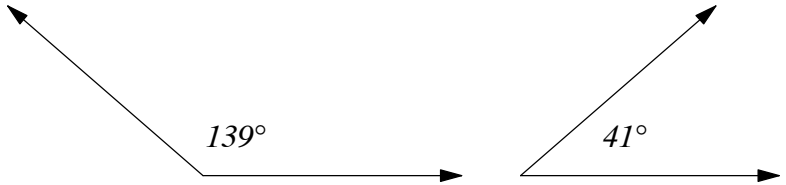
Figure 41 Complementary Angles

Supplementary Angles

Two angles are called supplementary angles if the sum of their degree measurements equal 180°. One of the supplementary angles is said to be the supplement of the other.

Example:

These two angles are supplementary.



Note that these two angles can be “pasted” together to form a straight line.

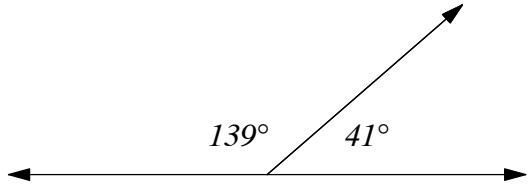


Figure 42 Supplementary Angles

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Vertical Angles

For any two lines that meet, such as in the diagram below, angle AEB and angle DEC are called vertical angles. Vertical angles have the same degree measurement.

Angle BEC and angle AED are also vertical angles.

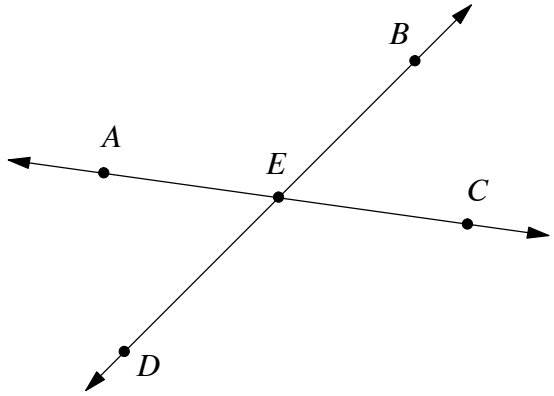


Figure 43 Vertical Angles

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Alternate Interior Angles

For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle D are called alternate interior angles. Alternate interior angles have the same degree measurement.

Angle B and angle C are also alternate interior angles.

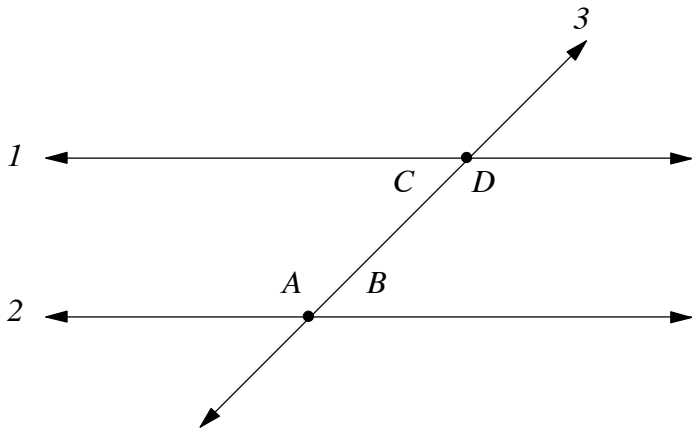


Figure 44 Alternate Interior Angles

Alternate Exterior Angles

For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle D are called alternate exterior angles. Alternate exterior angles have the same degree measurement.

Angle B and angle C are also alternate exterior angles.

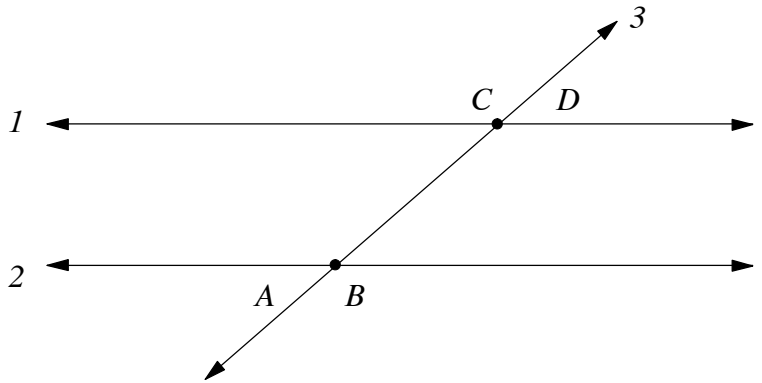


Figure 45 Alternate Exterior Angles

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Corresponding Angles

For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle C are called corresponding angles. Corresponding angles have the same degree measurement. Angle B and angle D are also corresponding angles.

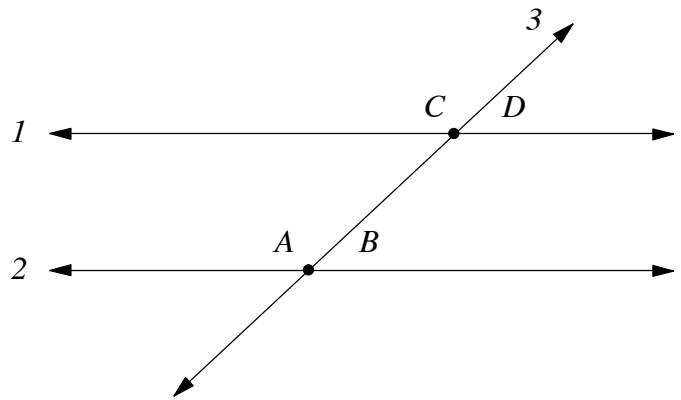


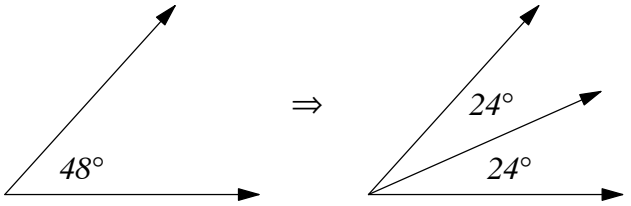
Figure 46 Corresponding Angles

Angle Bisector

An angle bisector is a ray that divides an angle into two equal angles.

Example:

The central ray on the right is the angle bisector of the angle on the left.



The centre ray on the right is the angle bisector of the angle on the left.

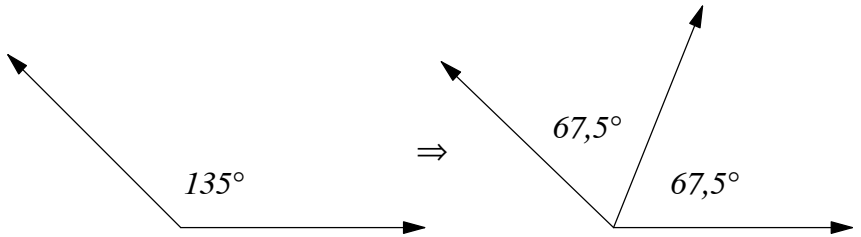


Figure 47 Angle Bisector

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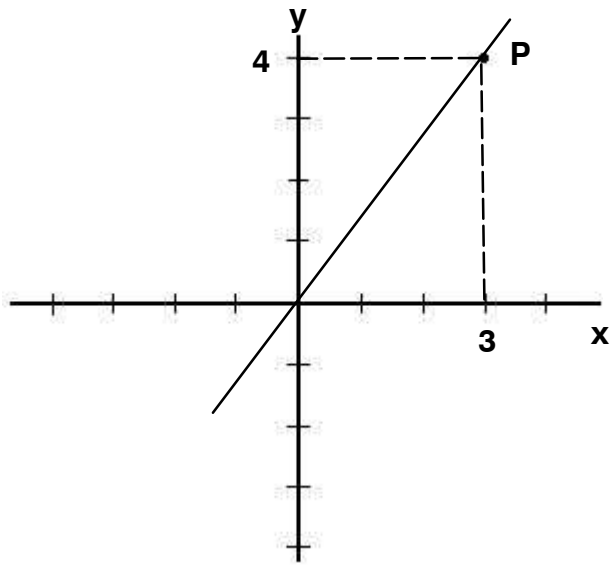
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Polar Coordinates

General

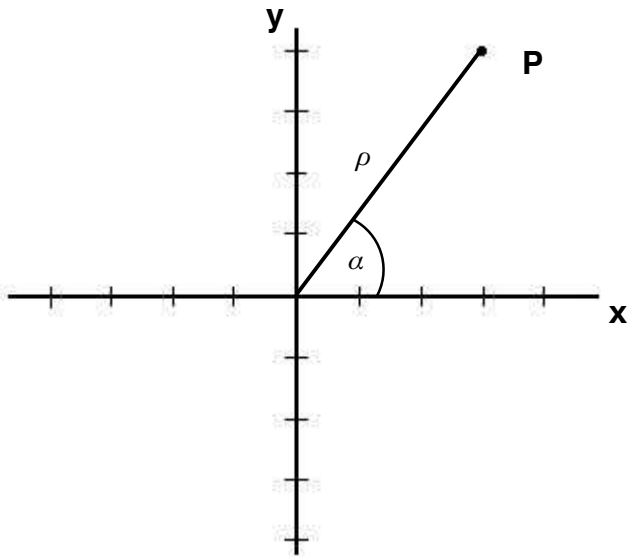
Polar coordinate systems are used in mathematics to define the position of a certain point in a two dimensional coordinate system.

To describe the position of a point polar coordinates use a different method than we already know. As mentioned in the lesson about coordinates & graphs a point will be defined by two coordinates. One coordinate in relation to the x – axis and one coordinate in relation to the y – axis. This system is called Cartesian coordinate system.



The coordinates of P are: $x = 3$ and $y = 4$
P (3;4)

In polar coordinate system the position of a certain point is defined by using the length of the line ρ connecting the point to the origin of coordinate axes and the angle α between the line and the x – axis.



The coordinates of point P are the length and the angle, also shown as P $(\rho;\alpha)$.

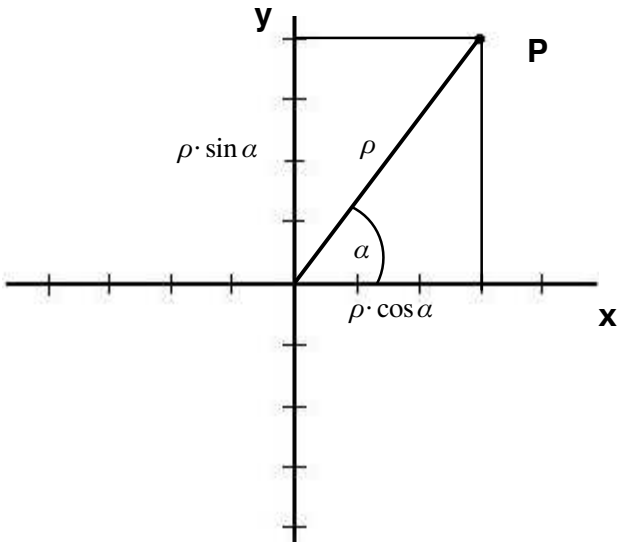
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Conversion from Polar coordinates into Cartesian coordinates

If Cartesian and Polar coordinate system use the same origin of coordinates and the same orientation of the axes Cartesian coordinates x and y can be calculated as follows:

$$x = \rho \cdot \cos\alpha$$
$$y = \rho \cdot \sin\alpha$$



Example:

Let us assume the length of ρ is 5 and the angle α is $51.13°$.

$$x = 5 \cdot \cos 53.13° = 3$$
$$y = 5 \cdot \sin 53.13° = 4$$

P (3;4)

Conversion from Cartesian coordinates into Polar coordinates

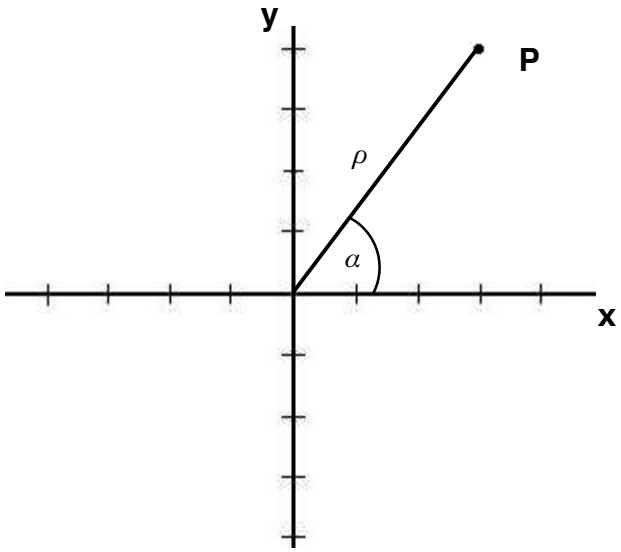
The conversion from Cartesian coordinates in Polar coordinates consist of two steps. In the first step the distance of point P from the origin of the coordinates is calculated (the length) and in the second step the angle α is calculated.

But let us start with the calculation of the length ρ by using Pythagoras' theorem as follows:

$$\rho = \sqrt{x^2 + y^2}$$

The second part of the coordinates, the angle α is calculated by using the arcus tangent of x and y. Arcus tangent is the reversed trigonometric tangent function $(\arctan\frac{y}{x})$.

$$\alpha = \arctan\frac{y}{x}$$



Example:

$$\rho = \sqrt{3^2 + 4^2} = \sqrt{25}$$

$$\rho = 5 \qquad \alpha = \arctan\frac{4}{3} = 53.13° \qquad P(5; 53.13°)$$

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